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TULTECH

Modelling Dispersion Coefficient in Meandering Channels by Use of Dimensional Analysis

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Abstract

Dispersion is the major means of determining pollutant transport in water bodies. Its effect is usually measured by dispersion coefficient, D, determined through tracer studies- a process that is cumbersome, time consuming and expensive. The parameter, D, is a most basic factor of measurement in hydraulic modeling of pollutants in rivers and its accuracy is important to measure the characteristic behavior of pollutants and preserve surface water quality. To circumvent the challenges of tracer studies, many researchers have used the geometric and hydraulic parameters of the river without considering the effect of meandering which is fundamental in river morphology. The aim of this research is to develop a robust dispersion coefficient model that accounts for the effect of bends in order to improve accuracy of the phenomenon. To do this, two out-door laboratory channels of 2 and 3 meanders respectively were constructed. Channel floor was covered with a layer of river sand and allowed to grow grass to perfectly mimick natural stream conditions. Tracer experiments were conducted on both channels at different velocities to obtain data for measured dispersion coefficient determined by Levenspiel and Smith method. Dimensional analysis was used to relate all the geometric and hydraulic parameters of the channels and MATLAB was used for calibration. Two models that include two new parameters namely, number of meanders (N) and ratio of radius of curvature to hydraulic radius ((R_c/R_h) were developed, one for each of the channels. Predicted results by both models were compared with the measured and with those obtained from some existing models. Statistical measures of accuracy namely, RMSE; MAE; DR and percentage error, showed that the new models performed better. The improvement in the new models is result of inclusion of the two new terms that reflect bend characteristics. The implication is that incorporating parameters of bend geometry in dispersion coefficient models improves their accuracy in determining the phenomenon in meandering channels. In conclusion, it is now known that bend parameters increase accuracy of D. New models to be used in determining dispersion coefficient in meandering channels have been developed, implying that a reliable and more accurate prediction of the dispersive ability of channels with bends is now possible. This will ensure better and reliable designs of treatment plants, better water resources management and pollution control that will improve surface water quality in compliance with United Nations development goals for cleanwater and sanitation for all, and preserve aquatic communities throughout the river reach.

Keywords: *Bend effect, dispersion coefficient, meandering, modeling, clean water, out-door laboratory.*

INTRODUCTION

Some of the global challenges in environmental engineering is the preservation of water purity and management of wastewater through treatment facilities that target domestic, industrial and agricultural uses of water. Water bodies are the most convenient and vulnerable means of waste disposal globally. The effect of such wastes on the water bodies are seen in oxygen depletion and high pollutant concentration leading to imbalance in aquatic ecosystem and results in aquatic kill and contamination of food chain and the consequent health implications on human.

Fortunately flowing water regimes have the ability to spread out and dilute such pollutants and, with time, recover their purity [1]. This dispersive characteristic is measured by a parameter called dispersion coefficient, D. This parameter is vital in the prediction of pollutant concentrations in natural rivers and is obtained through tracer studies which is expensive, time consuming and tedious. Several researchers have made considerable efforts to determine this parameter in much easier and most economically viable way by using the geometric and hydraulic features of the rivers or streams. However, there are still disparities between measured and predicted values of D because the factors that affect it are not yet completely known [2]. One such factor is effect of bends on D. Most of the models developed so far did not consider the effect of bend owing to the difficulty in measuring some parameters that are peculiar to it. Meandering is fundamental characteristic of rivers and streams and critical to their physical stability. Most rivers in the world have bends which introduce flow patterns different from that of straight courses. For example, Secondary flows occur at river bends with centrifugal and pressure gradient that disrupt equilibrium. Therefore models developed through the one dimensional dispersion equation (ADE) or by empirical models that do not include bend effect, will obviously not suffice for meandering cases.

Accurate determination of D is important to the engineer for the design of treatment units and water resources management [3]. The implication of poorly designed treatment units and poor water resources management include rise in global water shortages, higher pollution levels of water bodies thereby increasing the out-break of communicable and water-borne diseases that can be fatal; poor sanitary conditions when treatment facilities fail and the consequent increase in cost of water treatment; and loss of life. Accurate determination of D will raise compliance with water quality millennium goals by protecting surface water quality. Unfortunately, most models of D in literature were developed for straight channels. The aim of this work is to model dispersion in rivers with bends in order to obtain equations that will more accurately evaluate dispersion coefficient of rivers taking into account bend effect. This will be achieved by incorporating into the model, certain bend parameters like radius of curvature and number of meanders. The new models should improve design accuracy that will lead to better surface water purity and security.

Dispersion is the predominant transport mechanism in the mixing and dilution of pollutants in streams and rivers. Its effect is usually measured by dispersion coefficient (D), which is obtained through tracer studies known to be an expensive, time consuming and rigorous procedure. More recently, geometric and hydraulic characteristics of the river or channel have gained acceptance as means of determining D. This parameter is of vital importance in the design of treatment units, design of intake works and spill modeling. Dispersion remains the major means of defining pollutant transport in rivers and streams [4-8].

LITERATURE REVIEW

In this research, flows through open channel bend, mechanism of dispersion, and recent developments in dispersion coefficient models are reviewed.

Flow through open channel bends

Open channel flow around bends is a common occurrence observed in artificial and natural channel systems in the practice of hydraulic design. The common characteristics of such a flow include flow separation, transverse flows, loses in energy and variations in water surface mainly caused by the curvature of the bend. Secondary flow is known to enhance velocity which in turn, together with the meander effects, enhance hydraulic mixing of pollutants.

The most influential factor of flow through bends of open channels is the centrifugal force causing transverse acceleration on one hand and longitudinal flow deceleration on the other. Mixing is enhanced at every bend because of flow inversions.

At the entry of flow into a bend, the centrifugal force which emanates from the curvature of the channel produces a transversal slope in the surface of the water. The interaction between centrifugal force and transverse pressure gradient causes secondary flows in the cross-section and the secondary flows spread further by moving along the bend. Hence, at the bends, these processes lead to longitudinal velocity increase in the outer wall and decrease in the inner wall [9]. Transverse mixing arising from the centrifugal force and shear velocity exerts large influence on the rate of longitudinal dispersion. While the centrifugal force creates secondary flows that enhance transverse mixing that adversely affects longitudinal dispersion in the bend, there is also a large cross-sectional velocity gradient that tends to increase flow-wise velocity and hence positively impact on longitudinal dispersion [10, 11].

Factors that affect dispersion process

Several governing factors have been adduced to be responsible for the complexities in the dispersion process. These factors make the task of determining dispersion coefficient challenging. The relevant factors include channel geometry, flow velocity, shear velocity concentration of the pollutant, conservative properties of the dispersant [12], thermal stratification, bed roughness, dead zones [13], wind [14], Reynolds number, width-depth ratio and secondary current. Some of these factors are briefly discussed below:

Shear velocity

In shear flow, the velocity varies in the transverse direction, so that the parcels on different flow lines travel at different speeds, the faster ones overtaking the slower ones. Shear velocity is also called friction velocity. This means that it is a form by which shear stress may be expressed as velocity, see equation 1.

$$U_* = \sqrt{\frac{\tau}{\rho}} \qquad (1)$$

Where τ is shear stress and ρ is the density of fluid. Shear velocity is also given by equation 2:

$$U_* = \sqrt{ghs} \qquad (2)$$

Where g is gravity, h is hydraulic radius and s is the slope of channel.

The intensity of mixing in rivers is evaluated by longitudinal dispersion coefficient which is the effect of hydrodynamic parameters including the cross-sectional geometry and the variation of velocity across the channel [12, 15]. Channel shear velocity is one of the most important factors affecting the longitudinal dispersion coefficient. In most dispersion coefficient equations, shear velocity is presented as a ratio with mean flow velocity as a condition called frictional term

i.e $\frac{U}{U_*}$ = frictional term.

Frictional term is a factor of the roughness of the channel bed. [16] suggest that the most important parameter for accurate prediction of D is the $\left(\frac{U}{U*}\right)$ term.

Channel geometry

Width, depth and shape of channel affect dispersion. A channel of width larger than thirty meters disperses better than that of width less than ten meters [17]. In deep stratified water channels, velocity of flow varies from stratum to stratum and velocity is generally higher in deep waters than in shallow ones [18]. Rectangular or square laboratory channels give better mixing than those of trapezoidal or circular shapes [18].

Aspect ratio, that is, depth to width ratio (H/B) affect dispersion coefficient D [13]. Theoretical D increases for given $({}^{tD}/_{H}2)$ in which t is sampling time and H is depth of channel. As aspect ratio increases to infinity, the steady state dispersion coefficient is about eight times larger than that obtained by neglecting the wall effect. Natural irregularities of channels strongly affect the rate of transverse (rather than vertical) mixing because they are capable of generating a wide range of transverse motion [19]. Not too recent laboratory observation by [20] have shown that for natural cross-section channel geometries, the value of dispersion coefficient can be as much as 150% greater than corresponding values obtained for regular section.

Flow velocity

The most important parameter that affects dispersion is perhaps the flow velocity (U), of the river or channel [21, 22]. All equations of dispersion coefficient from Taylor to the most recent have all incorporated flow velocity either as longitudinal flow velocity or as transverse shear velocity or both. For an infinitely wide two-dimensional flow, the cause of flow is variation of velocity from one surface to another. In streams, the primary cause of flow is the difference in velocity in the transverse direction.

Generally, velocity distribution in stream creates areas of high velocity and those of lower velocity than the mean. This non-uniformity in velocity distribution affects dispersion greatly. The higher the velocity distribution, the lower the degree of dispersion. Transverse shear velocity is considered to be more important than vertical [18].

Secondary Currents

Secondary flow is basically flow in the plane perpendicular to the primary flow direction. It tends to decrease longitudinal dispersion by enhancing transverse mixing. They are formed when "unequal forces generate velocity components in a direction transverse to the flow producing a circulation. Secondary currents in straight and non-circular channels are generated by turbulence that is related to the formation of sand ridges. These are generally induced by corners. Such corner-induced secondary currents are characterized by flow moving into the apex of the corner with a return flow moving away from the corner and along the channel boundaries. Secondary flows occur in curved open channel reaches where centrifugal forces and pressure gradients are unbalanced [23].

Secondary current around pronounced curvatures generally are responsible for large magnitude of transverse circulation combined with the principal longitudinal flow. Solute dispersion by secondary current may therefore not be adequately described by the streamline direction only as there is dispersion effect in the stream-wise direction that is considered much more effective than the transverse turbulent diffusion [24].

Owing to the geometric complexities caused by meanders and varying sectional shape, secondary flows develop in streams and have hydrodynamic effect on transverse mixing. Although the secondary flow is weak compared to the primary flow, the associated helical motion significantly enhances the rate of transverse mixing and the greater the transverse mixing, the less the longitudinal dispersion.

Width-to-depth ratio

Aspect ratio is defined as the ratio of the width of the channel in a plane normal to the flow direction, to the flow depth. Aspect ratio is narrow when its value is equal to 1 and wide when it is 12 [25]. Generally, the geometry of the channel (that is, length, width and depth) affects its dispersion capacity. The longer the channel the higher the length to width ratio (i.e L/W), the more the concentration distribution curve approaches Gaussian or normal curve.

The main cause of longitudinal dispersion in natural rivers is the large width-to-depth ratio. Longitudinal dispersion coefficient changes even in the same stream as the flow depth or water level changes. [18] and [20] showed that for a more natural cross-sectional geometry channel, D increases.

A general form of dispersion coefficient equation 3 as presented by several researchers is as follows:

$$D = \theta\left(\frac{U}{U_*}\right)\left(\frac{W}{H}\right)^{\mathrm{b}} \tag{3}$$

The impact of irregular variation on width-to-depth ratio on longitudinal dispersion coefficient increased its value by ten or more times [26]. The term $\left(\frac{B}{H}\right)$ should affect the longitudinal dispersion because the D is a result of the transverse difference of longitudinal velocity. Dispersion coefficient generally varies as width, depth, velocity of flow shear velocity and other geometric and hydraulic irregularities change.

Large width-depth ration as experienced in natural rivers results in vertically well mixed pollutant injection at the near field region long before it becomes transversely well mixed. Because this occurs in the short-lived region, vertical mixing in rivers is usually ignored.

Rutherford opined that transverse mixing coefficient is not influenced by channel aspect ratio because the overall trend of the data is horizontal.

Low value of $\frac{W}{H}$ means that the river is wide and shallow while at high $\frac{W}{H}$ value the river is narrow and deep. At narrow and deep channels, i.e at high $\frac{W}{H}$ secondary currents are strong, smoothening off the primary velocity profile resulting in decrease in longitudinal shear. With increasing $\frac{W}{H}$ value, the secondary currents become less pronounced leading to an increase in the rate of primary velocity shear. Transverse shear is less important in relatively small values of $\frac{W}{H}$, while it dominates the dispersion characteristics in increasing $\frac{W}{H}$ value. The implication of this is that different reaches or regimes would exist for low and high $\frac{W}{H}$ ratios. Transverse dispersion coefficient increases as aspect ratio increases:

Beyond [27] and [28] the use of mathematical models to address environmental and ecological problem is increasing, showing their role as a tool to improve understanding of ecosystem properties [29]. In estimating D, it is preferable to use equations that are based on the hydraulic and geometric properties which can easily be determined from numerical methods. Combining the linear one-dimensional flow and dispersion equations, [30] were the first in presenting such equation. Their model for Fraude number less than 0.5 is expressed by equation 4:

$$\mathrm{D} = 0.058\frac{HU}{S},\ \text{for } \mathrm{F_n} = 0.5 \tag{4}$$

Where s is the slope of energy line; F_n is Froude number; H, flow depth (m) and u, longitudinal velocity (m/s). Furthermore, [31] modified an initial formula by [15, 18, 32] into [33] by equation 5.

$$D = \frac{\alpha U^2 B^2}{AH} \tag{5}$$

α is shape factor that depends only on the shape of the channel cross-section and velocity distribution across the channel, excluding the size of the channel. U is the magnitude of flow velocity for infinitely wide two dimensional channels in which the ratio of area to width(A/B) = R and expressed on equation 6.

$$U = {}^{Q}/_{A} \tag{6}$$

where A is cross sectional area of channel (m^2), Q is rate of flow through the channel (m^3/s). Substituting in equation 5, gives equation 7.

$$D = \frac{\alpha Q^2}{U_* R^3} \tag{7}$$

Where U_* is the transverse velocity (m/s). Liu found the value of α experimentally corresponds to equation 8 or 9

$$\alpha = 0.18 \left(\frac{U*}{U}\right)1.5 \tag{8}$$

Or

$$\alpha = 0.5 \left(\frac{U*}{U}\right)2 \tag{9}$$

The original equation 10a obtained by Taylor was as follows:

$$d = \frac{D}{UL} \tag{10a}$$

Substituting equation 8 or 9 into equation 10a, Liu obtained equation 10b.

$$d = \frac{0.168(\theta)^{0.25}B+2H)^{3.25}}{(LWB)^{1.25}} \tag{10b}$$

Where d is dimensionless coefficient called dispersion number, θ = detention time (sec); W is width of channel (m); L, length of channel (m).

The following recent researches considered the effect of lateral velocity gradient.

[34] used laboratory data to derive their equation to predict dispersion coefficient as equation 11.

$$\frac{D}{HU*} = 2 \left(\frac{B}{H}\right)1.5 \tag{11}$$

[12] used dimensional and regression analysis for the one step Huber method using 59 data sets to arrive at equation 12.

$$D = 5.915 \left(\frac{B}{H}\right)0.62\left(\frac{U}{U*}\right)1.428(U*) \tag{12}$$

They stated that Liu's equation is generally in good agreement with the measurement data whereas [34] equation underestimates it in many cases.

[12] used 35 out of 59 data sets obtained from 26 rivers in the United States of America consisting of hydraulic and geometric characteristics in streams and applied dimensional and regression analysis to derive their model then used the remaining 24 sets to verify it. After comparing their models with some existing cases, they concluded that their models more accurately predicted dispersion coefficient than those developed by [28, 30, 31] and [35].

[3] used genetic algorithm to develop his model with 67 data sets of observation from 24 rivers in USA. The rivers were selected because they represented a wide range of geometric and flow characteristics. He used 45 data sets for the model development and 16 for verification of model. He compared his model with those of [12, 15, 26, 31, 35] and [16] found that his model was better than the above-mentioned models.

[36] calibrated their model with very high precision compared to commonly used and highly rated models [37]. They used 116 data sets obtained from over 50 rivers in the Uk and the US for their work. The rivers had aspect ratio between 20 and 100. [37] concluded that [36] model is best suited for channels of trapezoidal shape having width>15m and >250m. [36] concluded that their model can predict longitudinal dispersion coefficient well for rivers within aspect ratio (B/H) range of 20 and 100 (ie 20<B/H<100).

[38] using the original theory and equation proposed by [18] and applying Von Kannan's law derived equation of the form, see equation 13.

$$D = \phi \frac{U_* B^2}{H} \tag{13}$$

By applying regression analysis on 16 sets of field data, they obtained that Φ= 0.6, s33 equation 14.

$$D = \frac{0.6 U_* B^2}{H} \tag{14}$$

They compared their model with that of Fischer and concluded that theirs gave a much closer result to the measured data.

[26] considered transverse mixing coefficient in the derivation of their model for longitudinal dispersion coefficient and developed expressions for h, u and ε for Fischer's integral equation and predicted the dispersion coefficient as equation 15 and 16.

$$\frac{D}{HU_*} = \frac{0.15}{8\varepsilon ro} \left(\frac{B}{H}\right) 5/3 \left(\frac{U}{U_*}\right) 2 \tag{15}$$

$$\varepsilon_{ro} = 0.45 + \frac{1}{3520} \left(\frac{U}{U_*}\right) \left(\frac{B}{H}\right) 1.38 \tag{16}$$

Where ε_{ro} is the transverse mixing coefficient (m2/s), u = longitudinal velocity.

In applying equation (11), the width-to-depth ratio must be greater than 10. They showed that the derived equation containing the improved transverse mixing coefficient predicts the longitudinal dispersion coefficient of natural streams better than that of [12].

[35] used 81 data sets and by application of regression and dimensional analysis derived, see equation 17.

$$D = 10.612 HU \left(\frac{U}{U_*}\right) \tag{17}$$

By combining their equation 17 and that of Seo and Cheng by trial and error, they obtained the following equations 18 and 19.

$$D = \left[7.428 + 1.775 \left(\frac{W}{H}\right) 0.62 \left(\frac{U}{U*}\right) 0.572\right] HU \left(\frac{U}{U*}\right) \quad (18)$$

[16] applied genetic algorithm to 65 data sets and proposed.

$$\frac{D}{HU*} = 2 \left(\frac{B}{H}\right) 0.96 \left(\frac{U}{U*}\right) 1.25 \quad (19)$$

They confirmed that the most effective parameter for accurate prediction of the longitudinal dispersion coefficient is the $\left(\frac{U}{U*}\right)$ term, called friction factor. They further maintained that models by [12]; [33] ; and [35] perform well in estimating D values of less than 100 m^2/s.

[39] also used genetic algorithm and 85 field data to derive the following empirical equation 20.

$$D = 0.91Q + 9.44 \quad (20)$$

Where Q is the low discharge. Equation 20 is thought to have limited predictive capacity for fast-flowing mountainous stream or stream of very low discharge rate.

[40] used MS model tree to derive equation for dispersion coefficient which they claim out-performed other existing formulae. For a width-to-depth ratio less or equal to 30.6, they obtained by equation 21.

$$\frac{D}{HU*} = 2.75 \left(\frac{B}{H}\right) 0.78 \left(\frac{U}{U*}\right) 0.11 (\sigma) 04.04 \quad (21)$$

For width-to-depth greater than 30m, they also obtained by equation 22.

$$\frac{D}{HU*} = 8.6 \left(\frac{B}{H}\right) 0.61 \left(\frac{U}{U*}\right) 0.85 (\sigma) 1.78 \quad (22)$$

Other researchers have shown that laboratory data can also be used to model dispersion coefficient in the same manner as for data from natural sources [32, 41, 42].

[42] used 24m long flume, a portion of which was covered by model submerged vegetation which created two layers in the flume, to study dispersion in flow with submerged vegetation. Their equation 23 was expressed as follows:

$$\frac{D}{U_{*}\ H} = \beta \left(\frac{h}{H}\right) 3 \left(\frac{H-h}{H}\right) 5/2 + r \left(\frac{H-h}{H}\right) 5/2 \quad (23)$$

Where:

H= flow depth; (H-h) = thickness of the upper (fast) zone; β= 40β21 β_2, where β1= constant fraction of the total velocity difference, ΔU, and $\beta_2 = \frac{\Delta U}{U_*}$

[18] used 10 laboratory data sets and applied statistical methods that connected some hydraulic and geometric variables to develop the following equation 24.

$$D = 0.011 \frac{U^2 B^2}{HU_*} \quad (24)$$

[43] affirmed that the accuracy of D estimation could be improved by accounting for curvature and that aspect ratio showed greater input on D than friction factor.

Some of the most recent empirical models include those of [44] and [45].

Li et al [44] wrote equation 25

$$\frac{D}{HU_*} = 2.82\,(\frac{B}{H})^{3.7613}\,(\frac{U}{U_*})^{1.4713} \qquad (25)$$

Wang and Huai [45] wrote equation 26

$$\frac{D}{HU_*} = 17.648\,(\frac{B}{H})^{0.3619}\,(\frac{U}{U_*})^{1.16} \qquad (26)$$

[46] used various machine learning algorithms including GPR, SVR, M5P and RV to estimate D. They found that M5P machine learning algorithm gave the best result. They used M5P to formulate the model in equation 27 and 28 and claimed that the M5P models are better than those obtained from other machine learning algorithms and those obtained empirically. The main advantage of M5P models is their ability to provide practical mathematical formulae as in equations 27 and 28 which are highly applicable to D estimations.

$$\frac{D}{HU_*} = 1.6896\,(\frac{B}{H}) + 20.0124\,(\frac{U}{U_*}) + 393.3343 \qquad (27)$$

$$\frac{D}{HU_*} = 2.8759\,(\frac{B}{H}) + 181.7915\,(\frac{U}{U_*}) + 339.5557 \qquad (28)$$

[47] added a sine term to obtain an equation that includes curvature as equation 29:

$$U\text{-}\bar{U} = \frac{AU_*}{K}\,(Y' - 0.1)0.5 + B\sin 2\pi y' \qquad (29)$$

A and B are regression coefficients that are determined from experimental data. U and U* are mean flow and frictional velocities respectively; K, Von Karman coefficient; Ū= averaged vertical velocity. U'=U-Ū is the velocity deviation between mean flow and average vertical velocities; Y' = dimensionless vertical coordinate defined as y/d and d is the water depth.

[48] proposed a model by slightly modifying Mozafari's equation 29 and obtained equation 30.

$$D_L = \frac{d^2}{\varepsilon}\left\{-0.0258\left(a - 0.38\frac{U_*}{K}\right)^2 + 0.0778\left(\frac{U_*}{K}\right)^2\right\} \qquad (30)$$

At $a = 0.38\frac{U_*}{K}$ the maximum value of D_L is obtained by equation 31

$$D_{Lmax} = 0.0778\left(\frac{d^2}{\varepsilon}\right)\left(\frac{U_*}{K}\right)^2 \qquad (31)$$

The regression coefficient, a, is obtained by modifying equation 25 to include a log function as proposed by Van Karman. At $a = 0$, equation 30 can be used for straight channels.

METHODOLOGY

The materials used in carrying out this research include out-door channels constructed with sandcrete blocks, sodium chloride as tracer, current meter, steel tape, water borehole drilled for the purpose of the study and chemistry laboratory. Buckingham pii theorem method of dimensional analysis and MATLAB computer software were used in developing the models.

Study area

The study location is the engineering campus of Kenule Beeson Saro-wiwa Polytechnic, Bori. Bori is a town located in Khana local government area of Rivers State in southern Nigeria at latitude of 4o 40'34.64"N and longitude 7o 21'54.68"E as shown in figure 3 below.

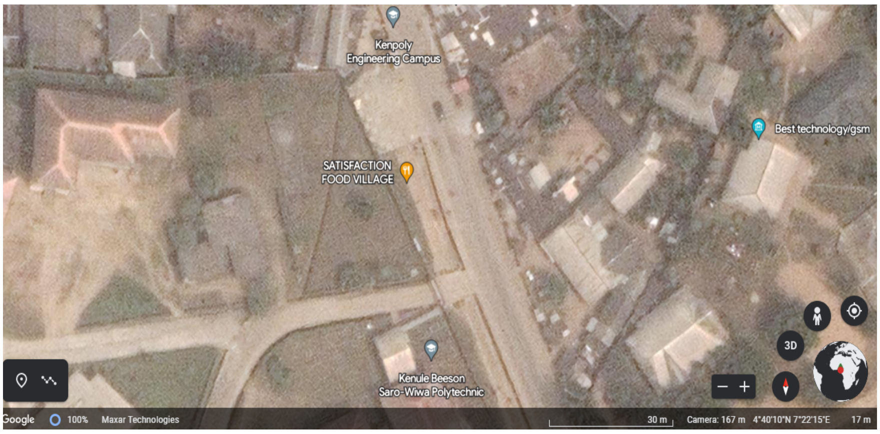

Figure 3. Google Map of Study Area

Experimental Setup

Two S shaped sinusoidal channels were made from the overall curvilinear lengths of $15.0m$. Both channels were constructed in approximately equal slope. Channel width was $0.2m$ for channel of 2 meanders and $0.4m$ for channel of 3 meanders while depth variations ranged between $0.2m$ *to* $0.3m$. The channel was allowed to develop for between 5 and 10 minutes and then uniform flow was maintained before the tracer was introduced and while samples were taken. Both channels were linked to a smaller reservoir (B) which supplied water to them through a $75mm\emptyset$ pipe fitted with ball valve to control flows. This also regulated velocity of flow once channel was fully developed. Water was supplied to the small reservoir by a larger tank (A) which has a borehole as source. Reservoir A when half full was made to fill reservoir B from which the channels were developed. This way constant head was maintained in both reservoir B and the channels. All three supply sources were run concurrently (i.e the borehole-tank A- tank B $-$channel) to maintain a constant head.

The channels were constructed of sandcrete walls and floor and sloped. The floor was covered with a layer of river sand. Grass and mold were allowed to grow on the floor and walls before the experiment were performed. These were meant to, as much as possible, mimic natural river conditions. The channels were models of the Kaani river stretch from Yeghe in Gokana local council (4039'35"N 7016'57"E to Wiiyaakare in Khana local council (4042'N 7021'E) all in the Nigeria's south-south state of Rivers. The flow velocity of the channels was varied arbitrarily to represent the frequent changes in velocity of the mimicked river. The study area experiences frequent rains between March and late September causing frequent variations in velocity. The sand mining activities upstream also fluctuates velocity. The data obtained in these experiments can therefore compare favorably with those of natural streams of similar features.

Salt solution was made by thoroughly mixing 40gramms of sodium chloride (common salt) with 200ml of water. This was used as the tracer material. This solution was introduced at 1.5m away from the channel feed point to reduce the effect of turbulence generated at the point of supply from tank B. Samples were collected at the channel outlet and titrated against silver nitrate solution according to Mohr's method (NITS). Sampling times were predetermined at regular intervals constituting constant-distance, variable-time method of sampling.

Measurement of velocity was by use of water current meter. Time measurement was by stop-watch. A meter tape was used in measuring water depth. Ten persons were lined up to take samples at the stipulated time intervals. Channel flow condition was determined by the Froude number given by equation 32.

$$\mathrm{Fr} = \frac{u}{\sqrt{gH}} \tag{32}$$

Where u is flow velocity, g is gravity and H is water depth of channel.

The highest velocity and water depth recorded for both channels were 0.791m/s and 0.088m respectively, both occurring in channel of 3 meanders. Highest Froude number in this work is thus 0.916 < 1. The flows in the experiments are subcritical.

Determination of longitudinal dispersion coefficient

The longitudinal dispersion coefficient (D) representing the actual or measured dispersion ability of the channel was determined by the Levenspiel and Smith method [49].

The procedure for computation of D by this method is as follows:

Determine the dispersion number (d) by equations 33 and 34.

$$d = \frac{1}{8}\left[\sqrt{8\sigma^2 + 1} - 1\right] \tag{33}$$

$$\text{Where } \sigma^2 = \frac{1}{T^2}\left[\frac{\sum C_i t_i}{\sum C_i}\right] - \left[\frac{\sum C_i t}{\sum C}\right] 2 \tag{34}$$

It is time taken after injection of tracer (in sec), C *is* tracer concentration at exit of channel (mg/l) as obtained from titration, T *is* average flow time (in sec) [50] which is actual detention time and is given by equation 35

$$T = \frac{\sum C_i t_i}{\sum C_i}, \tag{35}$$

Dispersion coefficient D was determined by D = udL

Where u = mean flow velocity; d = dispersion number and L = length of channel. Distance downstream from point source beyond the convective period is equivalent to equation 36 [11].

$$L \geq \frac{1.8 L_0^2}{RU} \tag{36}$$

Where L is distance from point source beyond the convective period (that is to the point where Taylor's one-dimensional dispersion equation begins to apply), Lo is characteristic width of channel, R is hydraulic radius (Area/wetted perimeter) and U is mean flow velocity.

Longitudinal dispersion model

Buckingham's pii theorem method of dimensional analysis was used to relate some parameters believed to affect dispersion coefficient and the equation derived is stated below as equation 37a and 37b [51].

$$D_L = \mathrm{HU}_* \left(\frac{U}{\mathrm{U}_*} \times \frac{B}{H} \times \frac{R_C}{R_H} \times N \times S_i\right) \tag{37a}$$

$$\therefore D_L = \varphi \mathrm{HU}_* \left(\frac{U}{\mathrm{U}_*}\right)\left(\frac{B}{H}\right)\left(\frac{R_C}{R_H}\right) N \times S_i \tag{37b}$$

Where D_L is the longitudinal dispersion coefficient, (m/s2); U_x, shear velocity (m/s); U_*, longitudinal velocity (m/s); B, width of channel; H, depth of flow (m); RC, radius of curvature (m); RH, hydraulic radius; N, number of meanders and Si is sinuosity. $\left(\frac{U}{U_*}\right)$ is called friction factor; $\left(\frac{B}{H}\right)$, is aspect ratio and $\left(\frac{R_C}{R_H}\right)$ is a new term developed in this model which reflects the effect of curvature and replaces bed shape factor, a parameter of the channel shape which is difficult to measure.

The statistical measures used for performance test as shown in table 6a and 6b below include

Root mean square error calculated by using equation 38

$$\mathrm{RMSE} = \frac{1}{\mathrm{N}}\sqrt{\sum(D_P - D_M)^2} \tag{38}$$

Where N is number of samples, Dp and Dm are predicted and measured values of dispersion coefficient. Mean absolute error (MAE), Discrepancy ratio (DR) and total effect of discrepancy ratio (DRS) has been respectively calculated by using equations 39, 40 and 41.

$$\mathrm{MAE} = \sum \frac{\mathrm{D}_P - \mathrm{D}_M}{\mathrm{N}} \tag{39}$$

$$\mathrm{DR} = \log\left(\frac{\mathrm{D_p}}{\mathrm{D_m}}\right) \tag{40}$$

$$\mathrm{DRS} =, \left(\sum_{i=1}^{N} \log\left(\frac{D_{xpi}}{D_{xmi}}\right)\right) / \mathrm{N}. \tag{41}$$

Accuracy is measured by the number of DR values that fall in the range between -0.3 and 0.3 % $Accuracy = \frac{1}{\mathrm{N}}\sum(-0.3 \le \mathrm{DR} \le 0.3)\, x100$

RESULTS AND DISCUSSION

The results obtained from the experiments conducted in the research are presented and discussed in this section. The measured dispersion coefficient and field data are presented in the below tables. Generated model data from experimental results of channel of 2 and 3 meanders are presented as shown in table 4. The presented results are used to develop the models in equations 42 and 43 using MATLAB

Measured dispersion coefficient and field data

Table 1 shows experimental results for channels of 2 and 3 meanders. Ten experiments were conducted paying attention to the longitudinal dispersion coefficient (*D*), shear velocity (*U*x), mean velocity(*U*), radius of curvature (*R*c), width of channel (*B*), depth of channel (*H*), hydraulic radius (RH), number of meanders (N), and sinuosity (Si) and their various results entered as presented below.

The models indicate the relationship between dispersion coefficient and the hydraulic and geometric parameters that affect dispersion in rivers. They show better prediction ability than the most recent models adjudged as the most reliable including sahay [3], Seo and Cheong [12], and Zeng and Huai [36], as is evident in their coefficient of correlation values and other statistical measures in table 6. Peculiar to this model is the inclusion of number of meanders (N), channel sinuosity (Si) and ratio of radius of curvature to hydraulic radius, parameters which define curvature. The implication of this is that curvature parameters enhance the accuracy of models of dispersion coefficient for rivers. Therefore models which do not contain bend parameters should not be used for calculation of D in curved channels. This statement is supported by the model of Sahay [3], which incorporated sinuosity and so fared better than the other two as seen in table 6. This means that empirical models are effective in similar flows and channel characteristics as those used in their calibration. A model might thus perform well when tested against a certain data set and fail to perform when tested against another data set of different flow characteristics and geometric configuration. It should be noted that empirical data are commonly known to contain missing values and consequently affects the performance of empirical models. Precisely, missing values are among the various challenges occurring in real-world data [52].

Equations 43 and 44 were specifically developed for use in channels/rivers with bends for which models developed for straight channels are not applicable or yield poor results. This means that data to be applied on them must contain N, R_C, S_I and R_H. If these data are not available, each of these variables can be assigned numerical value of 1 and used as for straight channels. This has been tried on a different set of data generated for waste stabilization pond design and it was found that the model for channel of 2 meanders performed well as design equation for waste stabilization ponds. This is the subject of another paper that we are writing.

Table 1 shows the result of measured data obtained from the experiments performed in the out-door channels. The results are used to prepare the log table for use as input data for regression analysis in MATLAB software. The table includes dispersion coefficient calculated as explained in section "Determination of longitudinal dispersion coefficient", longitudinal and shear velocities, width of channel, depth of flow, radius of curvature, hydraulic radius, number of meanders and sinuosity. The width and radius of curvature of each channel were constant and depth of flow varied with velocity. The number of meanders indicate the number of bends in each channel. Result shows that dispersion coefficient generally increased as velocity increased. The implication is that velocity is directly proportional to dispersion coefficient as indicated in the formulated equations 43 and 44. This is the general report of all dispersion coefficient equations in literature.

Table 1. Experimental results for channels of 2 and 3 meanders

Experimental results for Channel of 2 meanders									
Expt.	**D (m/s2)**	**U(m/)**	**U_* (m/s)**	**B(m)**	**H(m)**	**Rc**	**RH**	**N**	**Si**
1	0.22867	0.231	0.043	0.2	0.028	2.25	0.0219	2	1.53
2	0.14925	0.252	0.041	0.2	0.036	2.25	0.0265	2	1.53
3	0.45732	0.311	0.043	0.2	0.042	2.25	0.0396	2	1.53
4	0.51615	0.401	0.044	0.2	0.055	2.25	0.0324	2	1.53

5	0.44446	0.365	0.041	0.2	0.048	2.25	0.0355	2	1.53
6	0.64146	0.482	0.045	0.2	0.068	2.25	0.041	2	1.53
7	0.60265	0.563	0.047	0.2	0.078	2.25	0.044	2	1.53
8	0.54451	0.44	0.045	0.2	0.061	2.25	0.038	2	1.53
9	0.78204	0.63	0.049	0.2	0.082	2.25	0.0451	2	1.53
10	0.90693	0.71	0.051	0.2	0.084	2.25	0.0471	2	1.53
Experimental results for Channel of 3 meanders									
1	0.12597	0.212	0.047	0.4	0.02	1.85	0.016	3	1.65
2	0.25158	0.261	0.042	0.4	0.034	1.85	0.025	3	1.65
3	0.38032	0.388	0.046	0.4	0.045	1.85	0.031	3	1.65
4	0.45961	0.452	0.046	0.4	0.06	1.85	0.038	3	1.65
5	0.38553	0.413	0.037	0.4	0.052	1.85	0.052	3	1.65
6	0.82692	0.686	0.052	0.4	0.078	1.85	0.044	3	1.65
7	0.63678	0.639	0.05	0.4	0.084	1.85	0.046	3	1.65
8	0.40172	0.466	0.047	0.4	0.05	1.85	0.037	3	1.65
9	0.53074	0.587	0.051	0.4	0.069	1.85	0.039	3	1.65
10	0.65603	0.791	0.056	0.4	0.088	1.85	0.044	3	1.65

Table 2 shows the result of generated model data from experimental results of channels of 2 and 3 meanders. The obtained data are used to develop the multiple linear regression data and model for channels as presented in table 3 below. The values of log (U/U*), log (B/H), log(RC/RH) and log (D/HU*NSi) of this table are used as input data presented as x_1, x_2, x_3 and Y of table 3. These are the basic data used in MATLAB to develop the multi linear regression table as shown in table 3 below.

Table 2. Generated model data from experimental results of channel of 2 & 3 meanders

Generated model data from experimental results of channel of 2 meanders								
S/N	**U/U***	**B/H**	**RC/RH**	**D/HU*NSi**	**log(U/U)**	**log(B/)**	**log(RC/RH)**	**log(D/HU*NSi)**
1	5.3721	7.1429	102.7397	62.0676	1.6812	1.9661	4.6322	4.1282
2	6.1463	5.5556	84.9057	33.0451	1.8159	1.7148	4.4415	3.4979
3	7.2326	4.7619	56.8182	82.7525	1.9786	1.5606	4.0399	4.4159
4	9.1136	3.6364	69.4444	69.7006	2.2098	1.2910	4.2405	4.2442
5	8.9024	4.1667	63.3803	73.8042	2.1863	1.4271	4.1492	4.3014
6	10.7111	2.9412	54.8780	68.5057	2.3713	1.0788	4.0051	4.2269
7	11.9787	2.5641	51.1364	53.7221	2.4831	0.9416	3.9345	3.9838
8	9.7778	3.2787	59.2105	64.8253	2.2801	1.1874	4.0811	4.1717
9	12.8571	2.4390	49.8891	63.6058	2.5539	0.8916	3.9098	4.1527
10	13.9216	2.2472	47.7707	65.2971	2.6334	0.8097	3.8664	4.1789

Generated model data from experimental results of channel of 3 meanders								
1	4.5106	10.000	115.6250	27.0737	1.5064	2.3026	4.7504	3.2986
2	6.2143	5.8824	74.0000	35.5912	1.8269	1.7720	4.3041	3.5721
3	8.4348	4.4444	59.6774	37.1166	2.1324	1.4917	4.0890	3.6141
4	9.8261	3.3333	48.6842	33.6415	2.2850	1.2040	3.8854	3.5158
5	11.1622	3.8462	35.5769	40.4805	2.4125	1.3471	3.5717	3.7008
6	13.1923	2.5641	42.0455	41.1872	2.5796	0.9416	3.7388	3.7181
7	12.7800	2.3810	40.2174	30.6290	2.5479	0.8675	3.6943	3.4219
8	9.9149	4.0000	50.0000	34.5338	2.2940	1.3863	3.9120	3.5419
9	11.5098	2.8986	47.4359	30.4688	2.4432	1.0642	3.8594	3.4167
10	14.1250	2.2727	42.0455	26.8936	2.6479	0.8210	3.7388	3.2919

Table 3 shows the result of multiplication of appropriate pairs of data from table 2. The result is used in the formation matrix by which the constants and powers in equations 43 and 44 were obtained.

Table 3. Multiple linear regression data and model for channel of 2 and 3 meander

Multiple linear regression data and model for channel of 2 meander													
S/NO	x_1	x_2	x_3	Y	$x_1{*}x_2$	$x_1{*}x_3$	$x_2{*}x_3$	$x_{1,2}$	$x_{2,2}$	$x_{3,2}$	$x_1{*}y$	$x_2{*}y$	$x_3{*}y$
1	1.6812	1.9661	4.6322	4.1282	3.3054	7.7877	9.1074	2.8264	3.8655	21.4573	6.9403	8.1165	19.1226
2	1.8159	1.7148	4.4415	3.4979	3.1139	8.0653	7.6163	3.2975	2.9405	19.7269	6.3518	5.9982	15.5359
3	1.9786	1.5606	4.0399	4.4159	3.0878	7.9933	6.3047	3.9149	2.4355	16.3208	8.7373	6.8915	17.8398
4	2.2098	1.291	4.2405	4.2442	2.8529	9.3707	5.4745	4.8832	1.6667	17.9818	9.3788	5.4793	17.9975
5	2.1863	1.4271	4.1492	4.3014	3.1201	9.0714	5.9213	4.7799	2.0366	17.2159	9.4042	6.1385	17.8474
6	2.3713	1.0788	4.0051	4.2269	2.5582	9.4973	4.3207	5.6231	1.1638	16.0408	10.0232	4.5600	16.9292
7	2.4831	0.9416	3.9345	3.9838	2.3381	9.7698	3.7047	6.1658	0.8866	15.4803	9.8922	3.7511	15.6743
8	2.2801	1.1874	4.0811	4.1717	2.7074	9.3053	4.8459	5.1989	1.4099	16.6554	9.5119	4.9535	17.0251
9	2.5539	0.8916	3.9098	4.1527	2.2771	9.9852	3.4860	6.5224	0.7950	15.2865	10.6056	3.7025	16.2362
10	2.6334	0.8097	3.8664	4.1789	2.1323	10.1818	3.1306	6.9348	0.6556	14.9490	11.0047	3.3837	16.1573
∑ =	22.1936	12.8687	41.3002	41.3016	27.4930	91.0278	53.9121	50.147	17.8558	171.115	91.8501	52.9747	170.3653
Multiple linear regression data and model for channel of 3 meanders													
1	1.5064	2.3026	4.7504	3.2986	3.4686	7.1560	10.9383	2.269	5.3020	22.566	4.9690	7.5954	15.669
2	1.8269	1.772	4.3041	3.5721	3.2373	7.8632	7.6269	3.338	3.1400	18.525	6.5259	6.3298	15.374
3	2.1324	1.4917	4.089	3.6141	3.1809	8.7194	6.0996	4.547	2.2252	16.719	7.7067	5.3912	14.778
4	2.285	1.204	3.8854	3.5158	2.7511	8.8781	4.6780	5.221	1.4496	15.096	8.0336	4.2330	13.660
5	2.4125	1.3471	3.5717	3.7008	3.2499	8.6167	4.8114	5.820	1.8147	12.757	8.9282	4.9853	13.218

6	2.5796	0.9416	3.7388	3.7181	2.4290	9.6446	3.5205	6.654	0.8866	13.978	9.5912	3.5010	13.901
7	2.5479	0.8675	3.6943	3.4219	2.2103	9.4127	3.2048	6.491	0.7526	13.647	8.7187	2.9685	12.641
8	2.294	1.3863	3.912	3.5419	3.1802	8.9741	5.4232	5.262	1.9218	15.303	8.1251	4.9101	13.855
9	2.4432	1.0642	3.8594	3.4167	2.6001	9.4293	4.1072	5.9692	1.1325	14.895	8.3477	3.636	13.1864
10	2.6479	0.821	3.7388	3.2919	2.1739	9.9000	3.0696	7.0114	0.6740	13.979	8.7166	2.702	12.3078
∑ =	22.6758	13.198	39.5439	35.0919	28.4812	88.5941	53.4793	52.5845	19.299	157.469	79.6627	46.252	138.594

Equation 37a can be rearranged as,

$$\frac{D}{HU_x NS} = \left(\frac{U}{U_*}\right)\left(\frac{B}{H}\right)\left(\frac{R_c}{R_H}\right) \quad (37c)$$

Let Y be the dependent (response) variable and X_1, X_2 and X_3 be the independent (predictor) variables. Linearizing equation 32c and 32d yields,

$$\log Y = \log ao + a1 \log\left(\frac{U}{U_*}\right) + a2\log\left(\frac{B}{H}\right) + a3\log\left(\frac{R_c}{R_H}\right) \quad (37d)$$

Where,

$X_1 = \left(\frac{U}{U_*}\right)$, $X2 = \left(\frac{B}{H}\right)$, $X3 = \left(\frac{R_c}{R_H}\right)$, $Y = \frac{D}{HU_* NS}$

To obtain Y, a_o, a_1, a_2, and a_3, value calibration is required. MATLAB is used.

The matrix formation is as follows:

$$\begin{vmatrix} N & X_1 & X_2 & X_3 \\ X_1 & X_1^2 & X_1,X_2 & X_1X_3 \\ X_2 & X_2^2 & X_1,X_2 & X_2X_3 \\ X_3 & X_3^2 & X_1,X_3 & X_2X_3 \end{vmatrix} \begin{vmatrix} \log a_0 \\ a_1 \\ a_2 \\ a_3 \end{vmatrix} = \begin{vmatrix} Y \\ X_1Y \\ X_2Y \\ X_3Y \end{vmatrix} \quad (42)$$

The values of the regression constants, a_o, a_1, a_2, and a_3 through MATLAB operation.

$$x_1 = \log\left(\frac{U}{U_*}\right), \quad x_2 = \log\left(\frac{B}{H}\right), \quad x_3 = \log\left(\frac{R_C}{R_H}\right) \text{ and } \quad y = \log\left(\frac{D_L}{HU_* NS_i}\right)$$

Substituting the summed values in Table 3 for 2 meanders into the developed matrix in Equation 42, we have:

$$\begin{pmatrix} 10 & 22.1936 & 12.8687 & 41.3002 \\ 22.1936 & 50.1468 & 27.493 & 91.0278 \\ 22.8687 & 27.493 & 17.8558 & 53.9121 \\ 41.3002 & 91.0278 & 53.9121 & 171.1140 \end{pmatrix} \begin{Bmatrix} \log a_o \\ a_1 \\ a_2 \\ a_3 \end{Bmatrix} = \begin{pmatrix} 41.3016 \\ 91.8501 \\ 52.9747 \\ 170.3653 \end{pmatrix}$$

From the computation of the above matrix equation using MATLAB computer software, the values of the constant coefficients were evaluated as $\log a_o = -3.0928$, $a_2 = 3.2431$ and $a_3 = -1.0581$. From the anti-natural logarithm (i.e. $a_o = e^{-3.0928}$), we have $a_o = 0.045377$

Therefore, the developed model is:

$$\frac{D_L}{HU_* NS_i} = 0.045377 \left(\frac{U}{U_*}\right)^{3.343} \left(\frac{B}{H}\right)^{3.2431} \left(\frac{R_C}{R_H}\right)^{-1.0581} \quad (43)$$

Substituting the summed values in Table 3 for 3 meanders into the developed matrix in equation 42, we have:

$$\begin{pmatrix} 10 & 22.6758 & 13.1980 & 39.5439 \\ 22.6758 & 52.5845 & 28.4812 & 88.5941 \\ 13.1980 & 28.4812 & 19.2990 & 53.4793 \\ 39.5439 & 88.5941 & 53.4793 & 157.4687 \end{pmatrix} \begin{Bmatrix} \log a_o \\ a_1 \\ a_2 \\ a_3 \end{Bmatrix} = \begin{pmatrix} 35.0919 \\ 79.6627 \\ 46.2529 \\ 138.5937 \end{pmatrix}$$

From the computation of the above matrix equation using MATLAB computer software, the values of the constant coefficients were evaluated as $\log a_o = 7.5372$, $a_1 = -0.4699$, $a_2 = 0.1551$ and $a_3 = -0.8009$. From the anti-natural logarithm (i.e. $a_o = e^{7.5372}$), we have $a_o = 1876.4924$
Therefore, the developed model is:

$$\frac{D_L}{HU_* NS_i} = 1876.4924 \left(\frac{U}{U_*}\right)^{-0.4669} \left(\frac{B}{H}\right)^{0.1551} \left(\frac{R_C}{R_H}\right)^{-0.8009} \quad (44)$$

Table 4 shows the result of predicting dispersion coefficient with equations (43) and (44). It can be observed that results of the predicted values fared well with measured. Worthy of note For channel of 2 meanders, rows 1, 3, 7 and 10 and for channel of 3 meanders, rows 1, 2, 3, 4, 6, 8 and 9

Table 4. Measured and Predicted dispersion coefficient for the channels

Expt.	D_m (m/s²) for 2 meanders	D_p (m/s²)for 2 Meanders	D_m (m/s²) for 3 meanders	D_p (m/s²) for 3 meanders
1	0.22867	0.20166	0.12597	0.13692
2	0.14925	0.20999	0.25158	0.23559
3	0.45732	0.41075	0.38032	0.3365
4	0.51615	0.40205	0.45961	0.47012
5	0.44446	0.51782s	0.38553	0.40572
6	0.64146	0.56237	0.82692	0.64953

7	0.60265	0.67621	0.63678	0.69936
8	0.54451	0.48816	0.40172	0.40136
9	0.78204	0.8195	0.53074	0.55599
10	0.90693	0.96951	0.65603	0.75008

The measured and predicted dispersion coefficient for channel of 2 meanders in table 4 above showed perfect fit in the tenth experiments with correlation coefficient of 0.91 and 0.97 respectively while that of channel of 3 meanders showed better correlation in the sixth and tenth experiments, respectively.

Figures 4 and 5 below show the results of regression of predicted and measured values of D for the developed models and those compared. The developed models for this work produced the line of best fit with correlation coefficient of 0.958 and 0.936 respectively for channels of 2 and 3 meanders, compared to the models of Zeng and Hual [36] 0.954 and 0.914; Sahey [3] 0.946 and 0.930; and Seo and Cheong [12] 0.942 and 0.926. The results are very close because the compared models contain only parameter of straight channels while the new models include data of curvature. Both sets therefore prove good for the channel shape they were developed for. It should be noted that coefficient of correlation does not measure accuracy; it is rather a measure of relationship.

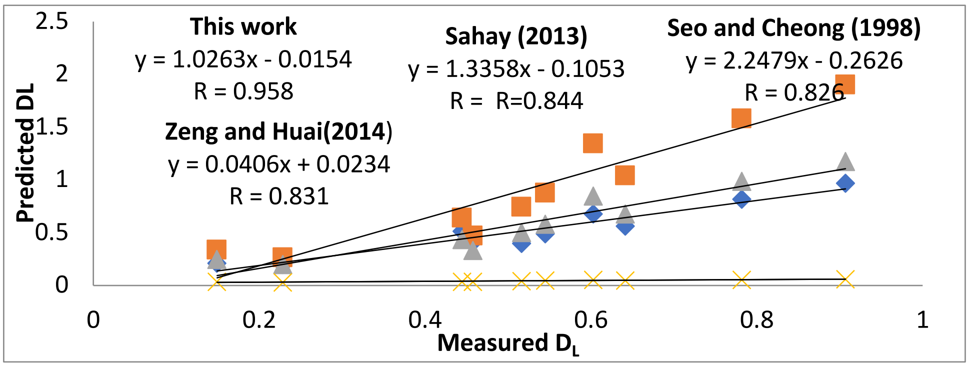

Figure 4: Compared predictions for channel of 2 meanders

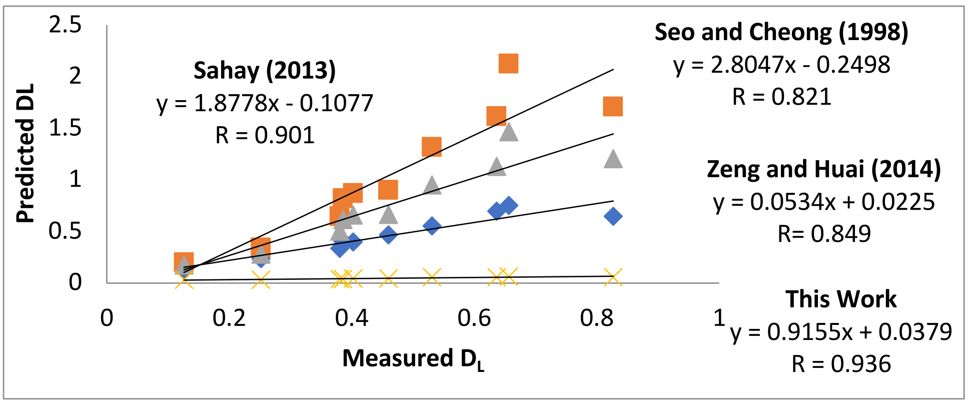

Figure 5: Compared predictions for channel of 3 meanders

Table 5 and 6 show the result of statistical measures of performance for the models. The table shows that the new models performed better in all fronts than those compared to them. The measures of accuracy shown in table 5a indicate that the new models predict D better than others as they have the lowest RMSE, MAE, DR and highest accuracy at values between -0.3 to 0.3. The new models were followed in accuracy by Sahay [3]. This is attributed to the inclusion of sinuosity which is a bend characteristic.

Table 5. Statistical measures of performances based on scholars

	RMSE		DR		MAE		-0.3 < DR < 0.3	
	M2	**M3**	**M2**	**M3**	**M2**	**M3**	**M2**	**M3**
Seo & Cheong	0.5 087	0.7219	0.2075	0.2859	0.39023	0.5653	70%	50%
Zeng & Huai	0.5246	0.5024	1.0304	0.6743	0.4887	0.4582	0%	30%
Sahay	0.1508	0.4786	0.1008	0.3334	0.12505	0.2788	100%	50%
This Work	0.0672	0.1676	0.0628	0.0647	0.06306	0.0387	100%	100%

Table 6. Correlation coefficients

Meanders	**Seo and Cheong**	**Zeng and Huai**	**Sahay**	**This Work**
	R	**R**	**R**	**R**
2 meanders	0.826	0.831	0.844	0.9579
3 meanders	0.821	0.849	0.901	0.9360

Effect of ratio-radius-curvature to hydraulic radius on dispersion

The coefficient of correlation between (R_c/R_h) and D are 0.8552 and 0.8108 for channels of 2 meanders and 3 meanders respectively as seen in figures 6 and 7. Implication is that the factor(R_c/R_h) affects dispersion as does aspect ratio. As (R_c/R_h) increases, D decreases. Curvature is the single most important factor that affects flow in meandering channels as it is the source of the helical motion that controls transversal mixing which in turn reduces longitudinal dispersion.

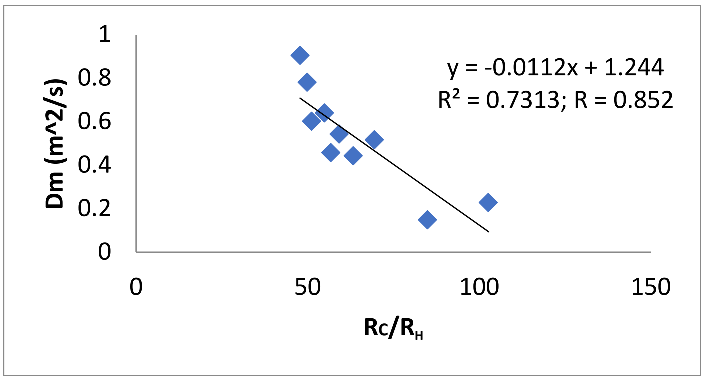

Figure 6. Graph of D against $\left(\frac{R_c}{R_H}\right)$for channel of 2 meanders

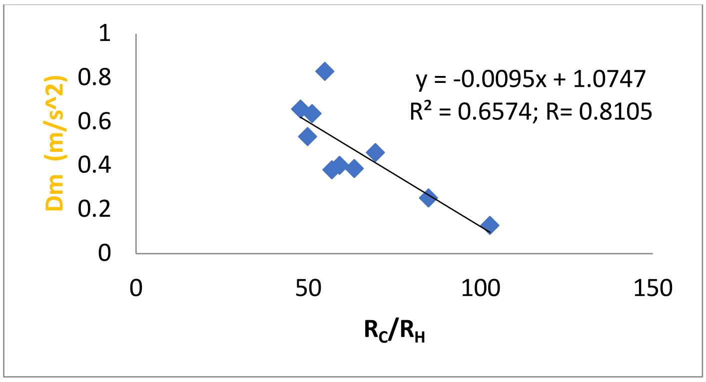

Figure 7. Graph of D against $\left(\frac{R_c}{R_H}\right)$ for channel of 3 meanders

Other parameters in all the models compared have been extensively studied by other researchers and their effects on D_L well determined and documented [19, 43, 45]. Their effect on D are not different in this work.

CONCLUSION

Much work has been done by researchers on dispersion coefficient using the factors explained in above sections, but none took cognizance of the effect of certain critical parameters of flow through bends such as number of meanders within the reach. Bends increase mixing of pollutants but reduce dispersion. Moreover, it induces centrifugal forces and pressure gradient that change flow signature from that of flows in straight courses. To preserve and maintain overall purity of the water body, it is important to include geometric parameters peculiar to bends of the river in dispersion coefficient models to account for dispersive ability of such regimes at such reaches.

Accuracy in the determination of dispersive ability of rivers is important for proper water resources management and design of wastewater treatment units and pollution control. To achieve this, certain flow and geometric characteristics of the regime such as bend characteristics must be included in the determining equations to account for flow in bends. For this reason, two new models of dispersion coefficient for meandering channels were developed using dimensional and regression analysis. Both were compared with the models developed by Seo and Cheong, Sahay, and Zeng and Huai which are some of the most recent models adjudged to be the most reliable in the prediction of dispersion coefficient for straight channels. The new models were found to be of higher accuracy than them all when subjected to statistical measures of accuracy including RMSE, MAE, DR and percentage error. The RMSR value of the new models for channels of 2 and 3 meanders were 0.0672 and 0.1676 respectively while Seo and Cheong model gave 0.5087 and 0.7219. Zeng and Huai was 0.524 and 0.5024 while Sahay showed 0.1508 and 0.4786. The values for MAE, DR, percentage error and correlation coefficient are in tables 6a and 6b. The improvement in the new model is attributed to the inclusion of two new parameters. First is number of meanders, (N), a hitherto unknown factor that affects dispersion. Second, is ratio of radius of curvature to hydraulic radius, $(R_c/R_{h)}$ and which showed strong relationship (R = 0.852 and 0.8105 for channels of 2 and 3 meanders respectively) with D, implying that it affects dispersion in meandering flows as do other regular terms in most dispersion coefficient equations such as aspect ratio and friction term. As a confirmation, the model by Sahay [3] which has sinuosity (a bend parameter) followed the new models in accuracy, further proving that bend features improve accuracy for D for curved channels.

Hydraulic radius which is easily measurable, can conveniently replace bed shape factor, a parameter that has proved very difficult to measure.

The new models performed better with a coefficient of correlation of 0.9579 against 0.831, 0.844 and 0.826 respectively for Zeng and Huai, Sahay and Seo and Cheong for channel of 2 meanders, while channel of 3 meanders showed a coefficient of correlation of 0.9360 against 0.853, 0.821 and 0.819 for Sahay, Seo and Cheong and Zeng and Huai models respectively. All 5 models showed strong relationship between predicted and measured dispersion coefficient. However, in terms of measures of accuracy, the new models showed better results in RMSE, MAE, Percentage error and discrepancy ratio. Thus including bend characteristics improves the accuracy of dispersion coefficient models for use in meandering channels. Better and reliable equations for D in channels with bends have been developed in this work. Improved designs for treatment plants, better pollution control and better water resources management that includes enhanced surface water purity, preservation of aquatic life in river bends can be achieved through these new models. This is one good step in progress towards achieving the United Nations sustainable development goals of access to clean water and sanitation for all.

CONFLICT OF INTERESTS

There are no conflicts of interest between the authors and his institutions that could appear to have influenced the work presented in this publication.

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