International Journal of Innovative Technology and Interdisciplinary Sciences
www.IJITIS.org
ISSN: 2613-7305
Volume 4, Issue 4, pp. 792-803, 2021
DOI: https://doi.org/10.15157/IJITIS.2021.4.4.792-803
Received July 25, 2021; Accepted November 17, 2021

Development of Optimal Transmission Rate of the Kinematic Chain by using Genetic Algorithms Coded in Mathcad

Anis Sulejmani[*a], Odhise Koça [b]

Department of Mechanical Engineering, Faculty of Mechanical Engineering,
Polytechnic University of Tirana, Albania

*[a]asulejmani@fim.edu.al; [b]okoca@fim.edu.al.

ABSTRACT

Part of machine element, require time to design, to calculate and to produce it. Production of helical gears passes through these processes. In this article is considered that production of these gears was made by milling machine. The goal is to minimize the time that needed to calculate the kinematic chain that is used to create the helical profile of gear. Kinematic chain contains the gears train with given set gears of the milling machine that mounted between driven shaft and raw part. Gear train usually is with two or three steps. An optimization algorithm will be used to find the correct tooth numbers of the gears train that gives the required transmission rate with acceptable accuracy. Genetic algorithm (GA) method has been used to find the optimal solution where the numbers of the teeth were variables and the transmission rate was the objective function.

Keywords: Gears train; helical gear; genetic algorithm, optimization

1. INTRODUCTION

Traditional metal cutting machines are often used in the production of machine elements such as helical gears or helical channels. In general, this way of production is very widespread as most of the production machines are metal cutting machines.

In this research work we have used a universal milling machine equipped with a gear train and an index plate. Index plate has transmission rate, generally i_{IP} = 40 [1]. Kinematic chain contains the gears train with given set gears of the milling machine that mounted between driven shaft and raw part. Gear train usually is with two or three steps. Furthermore, an optimization algorithm has been implemented to find the correct tooth numbers of the gears train that gives the required transmission rate with acceptable accuracy. Based on it, Genetic algorithm (GA) method has been used to find the optimal solution for calculation of the gear train [2-4].

2. HELICAL GEAR

In the Figure 1 is shown the geometry of the helical gear and its terminology [5-8].

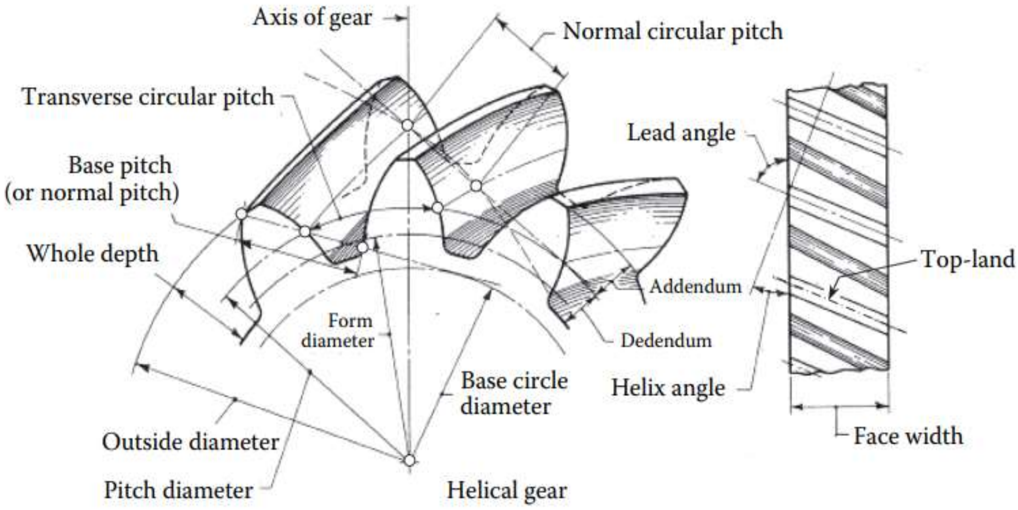

Figure 1. Helical gear geometry and terminology [5-8]

Below are shown the technical data of the helical gear to be produced, Table 1:

Table 1. Technical data of the helical gear to be produced

Term	Symbol	Value	Units
Module	m_n	6	mm
Tooth number	z	15	-
Pressure angle	α	20	degree
Helix angle	β	20	degree
Pitch diameter	d	95.776	mm
Outside diameter	d_a	107.776	mm
Root diameter	d_f	80.776	Mm
Face width	b	100	Mm

3. KINEMATIC RELATION BETWEEN MACHINE DRIVEN SHAFT – RAW GEAR

Figure 2 depict the helical line for a point of the pitch diameter and its elevation for a complete rotation. The relation between the route of the metal cutting machine, helix angle and the point in the pitch circle is given by Figure 3.

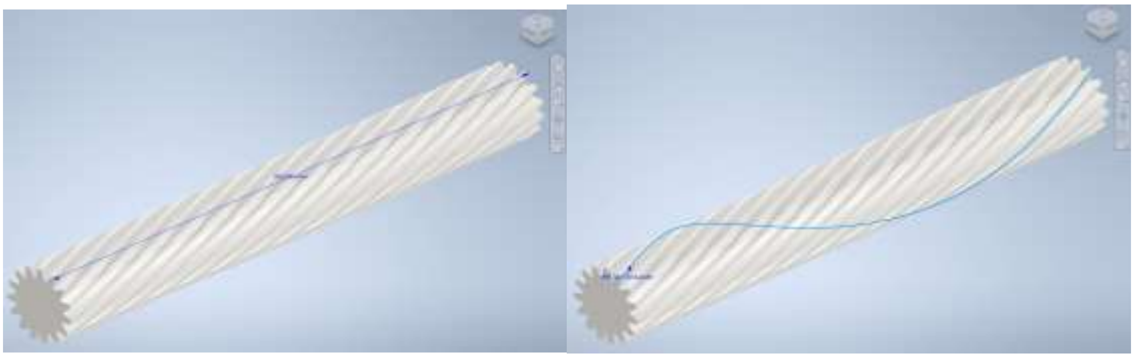

Figure 2. Helical gear geometry and terminology

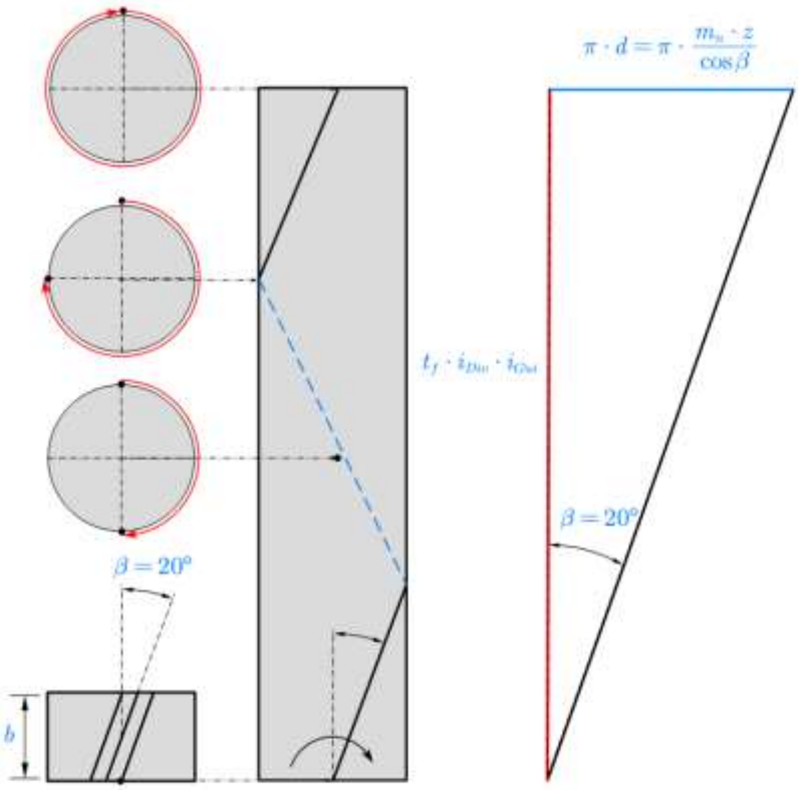

Figure 3. The triangle of the relation between the metal cutting machine and the gear

From the geometry we can received the equation (1):

$$\tan(\beta) = \frac{\frac{\pi \cdot m_n \cdot z}{\cos(\beta)}}{t_f \cdot i_{Div} \cdot i_{Gui}} \tag{1}$$

where t_f = 6 mm– milling machine step value, i_{Div} = 40 – index plate transmission ratio, i_{Gui} – the transmission ratio of the gear train that needs to be determined

From equation (1), to define the transmission ratio of the gear train, the general expression is derived as follows:

$$i_{Gui} = \frac{\pi \cdot m_n \cdot z}{t_f \cdot i_{Div} \cdot \sin(\beta)} \tag{2}$$

Based on the technical data of Table 1 and from the characteristics of the metal cutting machine we have the value that correspond to i_{Gui} = 3.6742.

4. GEAR TRAIN

This kinematic chain usually consists of two transmission ratios [1, 9, 11] as shown in Figure 4 and 5. The gears have a module equal to 1 mm and are taken from the machine set.

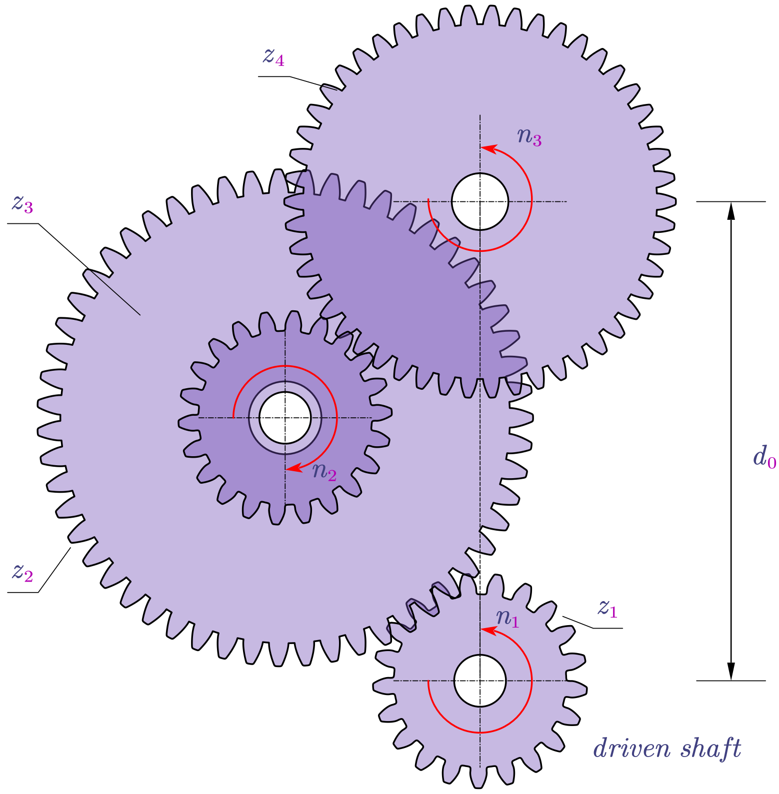

Figure 4. The gear train composition for a right-hand helical gear

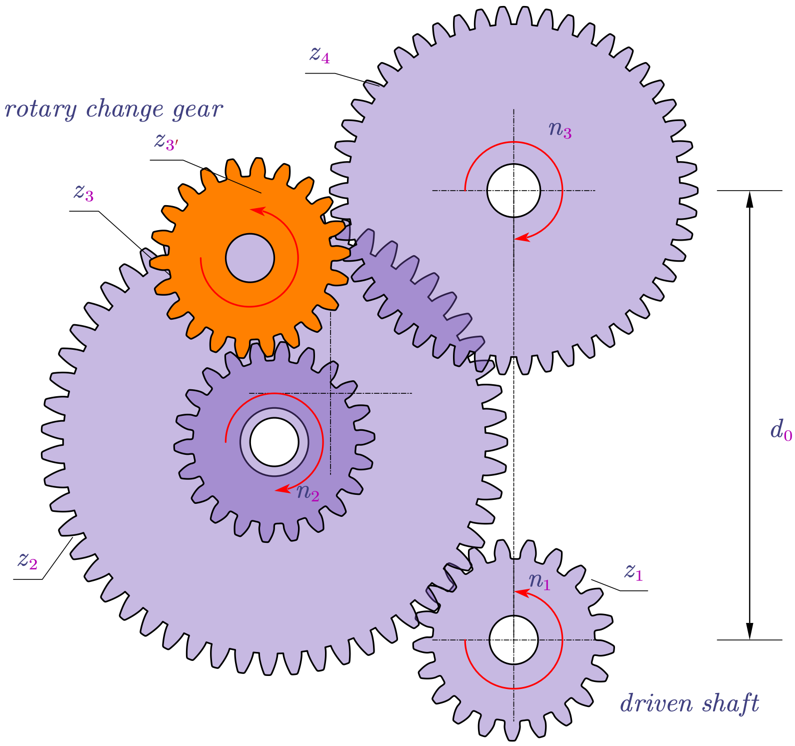

Figure 5. The gear train composition for a left-hand helical gear

The gear set is specific to the metal cutting machine. In the milling machine in this case, repetition of gear teeth number is possible. This set is given in Table 2.

Table 2. Gear set used in the gear train.

15	21	27	33	39	45	51	57	63	69	75	81	87	93	99
16	22	28	34	40	46	52	58	64	70	76	82	88	94	100
17	23	29	35	41	47	53	59	65	71	77	83	89	95	-
18	24	30	36	42	48	54	60	66	72	78	84	90	96	-
19	25	31	37	43	49	55	61	67	73	79	85	91	97	-
20	26	32	38	44	50	56	62	68	74	80	86	92	98	-

Based on Figure 4 i_{Gui} can be expressed as equation (3):

$$i_{Gui} = \frac{z_2}{z_1} \cdot \frac{z_4}{z_3} = 3.6742 \quad (3)$$

The method to calculate the number of teeth from this gear set is the Genetic Algorithm Method.

5. GENETIC ALGORITHM METHOD FOR CALCULATING THE NUMBER OF TEETH

5.1 Objective function

The objective function for this research work is written by using equation (4) [6]:

$$f_{obj}(z_1, z_2, z_3, z_4) = \left| \frac{z_2}{z_1} \cdot \frac{z_4}{z_3} - 3.6742 \right| \leq \varepsilon = 10^{-3} \quad (4)$$

where z_i – is the number of teeth taken from the technical data of the gear set.

5.1.1 Penalty function and geometry

There are some geometry conditions [6, 12-14] that needed to input as penalty function. One of them is that the sum of gear tooth should be larger than the equivalent tooth corresponding the distance of axes in equation (5).

$$\frac{d_0}{m} < \sum_{i=1}^{n} z_i < \frac{3 \cdot d_0}{m} \quad (5)$$

5.2 First generation data

The algorithm for inserting the data of the first generation, which are taken from the table of the gear set, is shown in Figure 6 [3, 9].

$$
\begin{array}{l}
DATA^{\langle 0\rangle} := \;\text{while}\; i_2 < N_k \\
\quad RD_{i_2,0} \leftarrow \;\text{for}\; i_1 \in 0..3 \\
\quad\quad\quad z_{i_1} \leftarrow Z_{trunc(rnd(rows(Z)))} \\
\quad\quad\quad i_1 \leftarrow i_1 + 1 \\
\quad\quad z \\
\quad x \leftarrow \;\; x \leftarrow 1 \;\text{ if }\; 50 < \sum_{k=0}^{3} \left(RD_{i_2,0}\right)_k < 150 \\
\quad\quad\quad x \leftarrow 0 \;\text{ otherwise} \\
\quad i_2 \leftarrow i_2 + x \\
\quad RD \\
RD
\end{array}
$$

Figure 6. Creation of the first generation with space $N_k = 20$, and limited by geometric condition

The *mutation ratio* and the *crossover ratio* are: $p_m = 10\%$ and $p_c = 50\%$, where p_m – is the *mutation ratio* and p_c – is the *crossover ratio* [1, 6].

5.3 Fitness function

After every set of data that, are used the following function coded in Mathcad to evaluate them all. The objective function used here is the objective function in equation 3. The calculation is given in Figure 7.

$$
\begin{array}{l}
FF(data, n) := \;\text{for}\; nn \in 1..N_k - 1 \\
\quad \text{for}\; i \in 0..N_k - 1 \\
\quad\quad Fi_i \leftarrow \dfrac{1}{1 + F_{ob}\left[\left(data^{\langle n\rangle}\right)_i\right]} \\
\quad\quad i \leftarrow i + 1 \\
\quad Tot \leftarrow \sum_{i=0}^{N_k - 1} Fi_i \\
\quad \text{for}\; m \in 0..N_k - 1 \\
\quad\quad P_m \leftarrow \dfrac{Fi_m}{Tot} \\
\quad\quad R_{m,n} \leftarrow rnd(1) \\
\quad C_0 \leftarrow P_0 \\
\quad C_{nn} \leftarrow P_{nn} + C_{nn-1} \\
\quad (R \;\; C)
\end{array}
$$

Figure 7. Fitness function

5.4 Selection function

In Figure 8 is given the selection program for data selection.

```
Selection(X,R,C) := | Index ← for i ∈ 0.. N_k − 1
                    |            | Index_i ← for j ∈ 0.. N_k − 2
                    |            |              | j + 1  if  C_j ≤ R_i ≤ C_j+1
                    |            |              | 0  if  0 ≤ R_i ≤ C_1
                    |            | i ← i + 1
                    |            | Index
                    |         for i ∈ 0.. N_k − 1
                    |           | A_i ← X_Index_i
                    |           | i ← i + 1
                    | A
```

Figure 8. Fitness function

5.5 Crossover function

A crossover function Figure 9 is used to determine a data set that converges closer to the specified objective. This crossover function is as below:

```
CrossOver(X,n) := | CP ← | P0 ← Parents(X)
                  |      | for i ∈ 0.. rows(P0) − 1
                  |      |   | CP_i,n ← (rnd(2))
                  |      |   | i ← i + 1
                  |      | ceil(CP^⟨n⟩)
                  | for i ∈ 0.. rows(P0) − 1
                  |   | P1_i ← augment(submatrix(P0_i,0,CP_i − 1,0,0)^T, submatrix(P0_i+1,CP_i,3,0,0)^T)^T  if i < rows(P0) − 1
                  |   | P1_i ← augment(submatrix(P0_i,0,CP_i − 1,0,0)^T, submatrix(P0_0,CP_i,3,0,0)^T)^T  otherwise
                  |   | i ← i + 1
                  | Temp2(X,P0,P1)
```

Figure 9. Crossover function

5.6 Mutation function

Lastly, we are going to use another function to further improve the results. This is the mutation function, which changes a parameter from the data set to get closer to the objective. This mutation function is shown in Figure 10.

```
Mutation(X) := | B ← | k ← 0
               |     | FP ← rows(X_0)·rows(X)
               |     | while k < ceil(pm·FP) − 1
               |     |     | R_k ← rnd(FP)
               |     |     | k ← k + 1
               |     | ceil(R)
               | A ← X
               | for j ∈ 0.. rows(B) − 1
               |   (A_floor((B_j − 1)/4))_mod(B_j − 1, 4) ← Z_trunc(rnd(rows(Z)))
               | A
```

Figure 10. Mutation function

5.7 Genetic algorithm

Figure 11 shows all the functions above composed together. The created space is 10^4 generations, which will be created first, then they will be evaluated by the objective function, which in our case is the transmission ratio of the gear train with an error acceptance equal to $\varepsilon = 10^{-3}$. If a parameter from the data set z_1, z_2, z_3, z_4 completes the criteria of the objective function, then this algorithm stops and shows the calculated result.

```
GA := | for i ∈ 0.. 10^4
      |   | DATA^<0>
      |   | Temp1 ← FF(DATA, i)
      |   | Temp2 ← Selection[DATA^<i>, (Temp1_0,0)^<i>, Temp1_0,1]
      |   | Temp3 ← CrossOver(Temp2, i)
      |   | DATA^<i+1> ← Mutation(Temp3)
      |   | i ← i + 1
      |   | TRUE ← | for k ∈ 0.. N_k − 1
      |   |        |   | TRUE ← 1  if (F_ob(DATA_k,i) ≤ ε)
      |   |        |   |        0  otherwise
      |   |        |   | k ← k + 1
      |   |        | TRUE
      |   | break if TRUE = 1
      | DATA
```

Figure 11. Genetic algorithm used to find number of teeth for a given transmission rate

6. RESULTS

This method is a converging method. This means that it is guaranteed to find a solution. After every attempt we get a valid solution which is different from the others because of the condition that the accepted error is equal to $\varepsilon = 10^{-3}$ and the iteration number is also different. Also, the solution time is much shorter with this method than calculating by hand. The graph of the objective function for all the solutions until the required result is given in Figure 11. The best solution from every generation and the trend of improvement after every step is given in Figure 12.

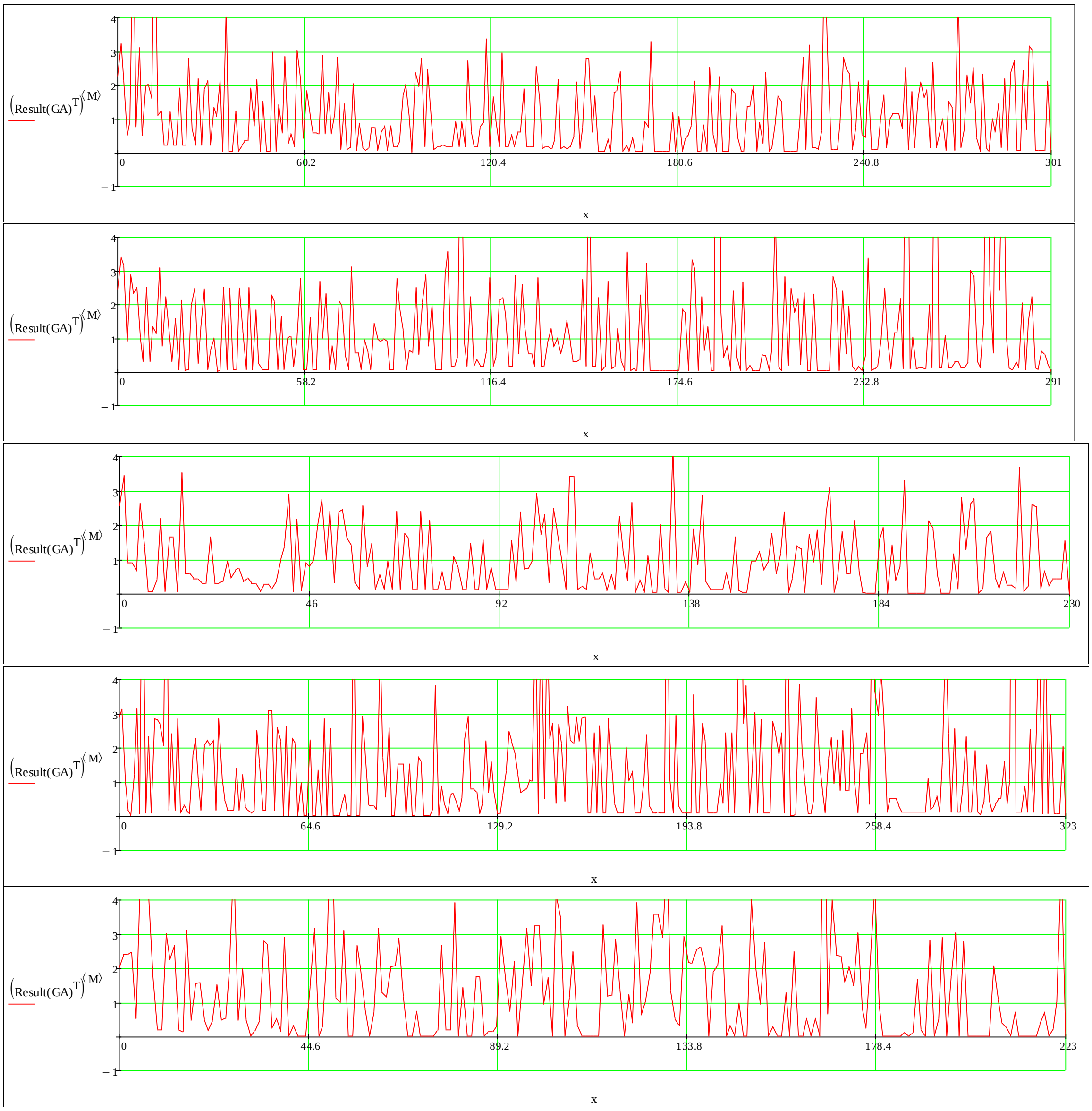

Figure 12. Objective function results for each step of genetic algorithm

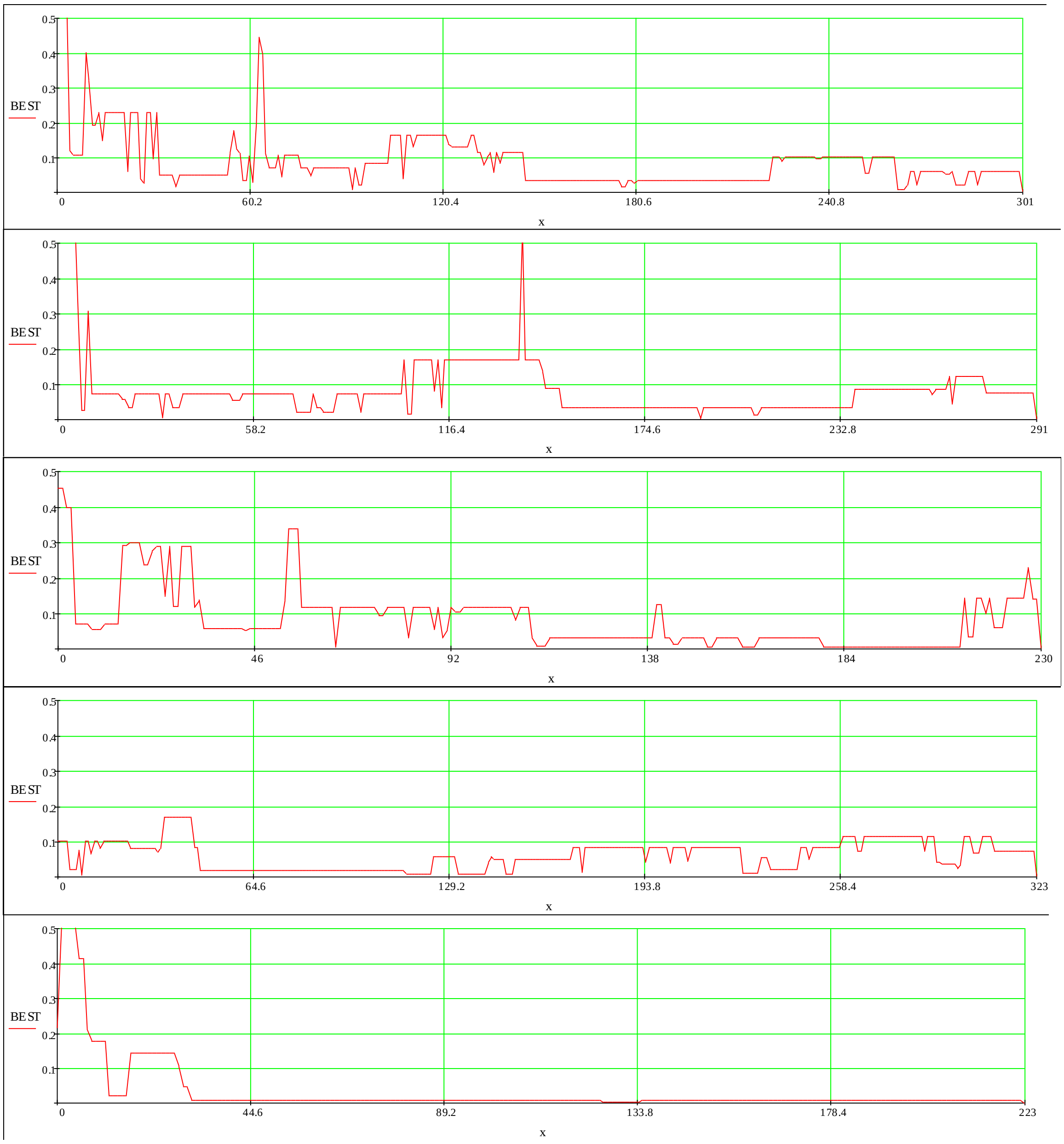

Figure 13. The best results for each generation

The solutions for the five cases shown in Figures 12 and 13 are listed in Table 3.

Table 3. Result of optimal transmission rate

Case	I	II	III	IV	V
z_1	54	42	22	20	43
z_2	58	67	59	36	90
z_3	19	33	54	24	49
z_4	65	76	74	49	86
i_{gui}	3.6745	3.6739	3.6751	3.6750	3.6735
i_{req}	3.6742	3.6742	3.6742	3.6742	3.6742
$\varepsilon = \lvert i_{req} - i_{gui} \rvert$	0.0003	0.0003	0.0009	0.0008	0.0007

7. CONCLUSION

Based on this research work following conclusions were drawn as follows:

- With this method we get a faster solution than the classical method for calculating this kind of problems.
- If the problem changes to using a bigger chain train with a much bigger transmission ratio, then using the classic method, the solution time changes exponentially bigger. This is solved very fast with the genetic algorithm method by changing the objective function and the data matrix of the first generation.
- Even if the database of the gear set changes for any reason (depends on metal cutting machine gear set, missing gears etc.), we can directly change the data in the first-generation table and keep everything else the same.
- For every transmission ratio needed, we can find a solution regardless of the number of iterations (solutions that depend on the starting data set).

CONFLICT OF INTERESTS

The authors would like to confirm that there is no conflict of interests associated with this publication and there is no financial fund for this work that can affect the research outcomes.

REFERENCES

[1] Jorgji V. (1983) Metodika e zgjedhjes së ingranazheve për kitarën e frezës, Tiranë.

[2] Bode K., Koça O., Sulejmani A., Genetic Algorithms for Optimum Machine Elements, *International Journal of Engineering Innovation & Research*, 2015; 4(1); 151 -157.

[3] Teklemarian T.R.D.G. and Senthil Kumar G.N.A. Optimization of gear parameters by using genetic algorithm. *International Journal of Mechanical Engineering*, 2014; 2(3); 55-62.

[4] Lin C.Y. and Hajela P. Genetic Algorithms in Optimization Problems with Discrete and Integer Design Variables, *Eng.*, *Opt.*, 1992; 19; 309-327.

[5] Radzevich S.P. (2021) Dudley's Handbook of Practical Gear Design and Manufacture, CRC Press, Garching, Germany.

[6] Arora J.S., Introduction to optimum design", Elsevier press, MA, USA.

[7] ISO 6336-1:2019. Calculation of load capacity of spur and helical gears — Part 1: Basic principles, introduction and general influence factors

[8] DIN 3990-1:1987-12. Tragfähigkeitsberechnung von Stirnrädern; Einführung und allgemeineEinflußfaktoren Publication date 1987-12.

[9] Spurs gear optimization by using genetic algorithm.Yallamanti Murali Mohan, T.Seshaiah Vol2 Issue 1, Jan-Feb. 2012, pp. 311- 318

[10] Koça O., Sulejmani A. and Alimhilli P. Direct method of rigidity using Mathcad Software, *IOP Conf. Ser.: Mater. Sci. Eng.*, 2021; 1024; 012093.

[11] Wittel H., Jannasch D., Voßiek J. and Spura C. (2019) Roloff/Matek Maschinenelemente: Normung, Berechnung, Gestaltung, Springer Fachmedien Wiesbaden, Germany.

[12] Budynas R., Nisbett K., (2019) Shigley's Mechanical Engineering Design, McGraw-Hill Education, USA.

[13] Koça, O., Sulejmani, A., & Dhoska, K. Pressure distribution on rolling-slide contact problem, *Pollack Periodica*, 2021; 16(1); 71-76.

[14] Koça O., Sulejmani A. and Lumi D. Equivalence of High Pairs on Mechanism. *International Journal of Mechanical Engineering and Technology*, 2020; 11(9); 48-55.