



Design of a Robust Controller Using Sliding Mode for Two Rotor Aero-Dynamic System

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ABSTRACT

This paper deals with two rotor aero-dynamic system (TRAS) which is a multi-input multi-output highly coupled nonlinear system, for creating a mathematical model based following the Lagrange's Equations, and creating controllers for Sliding Mode Control and Linear Quadratic Regulator. The Sliding Manifold is designed by employing the reduced order representation. Linear Quadratic Regulator has been created by linearizing the nonlinear system acquired by creating the state space representation following the mathematical model. The signal tracking conditions of PID, SMC, and LQR have been discussed. Although the proposed control methodology has perfect actuation time, the tracking efficiency was not satisfactory. Therefore, a rework on the parametrization and introduction of filters, i.e. types of Kalman Filters have been proposed as a conclusion.

Keywords: Helicopter, MIMO, LQR, SMC, TRAS, Lagrange's Equations, Lagrangian.

1. INTRODUCTION

Two Rotor Aero-dynamical system is a laboratory set up that demonstrates a high order coupled nonlinear multi input multi output system which can rotate freely in horizontal and vertical planes [4]. In this manner, the set up that has been developed by Inteco, resembles that of a helicopter.

Regarding the 'Two Rotor Aero-dynamical System' and similar setups, researchers have been using them to model helicopter like structures to prevent paying astronomical prices for real life failures. As the phase of control theory has switched from classical to modern and advancements in technology brought out more complicated designs for aerospace industry, the number of researches done with this lab equipment has increased.

During the times of classical control, PID controllers were the generic option. Versatility, ease of implementation and ability to create variations such as PI and PD, this controller is also studied for the modern structures [8]. However, with the introduction of modern control the interest has been shifted to nonlinear control structures that could govern advanced dynamics.

Regarding the mathematical model for the system, followed by [22], [23], and [24], Lagrange's Equations have been used. Moreover, for the modelling part [4], [5], and [6] have been primary resources.

For the design of the controllers, [9], [12], [13], [14], and [22] have created the understanding for the Sliding Mode Control whereas, [10], [11], and [28] are the corner

stones for Feedback Linearization and Linear Quadratic Regulator. Further references on the issue are referred to on [31], [32], [33], and [34].

2. MATHEMATICAL MODELLING

Mathematical modelling of the system as in analytical dynamics by implementing the Lagrange's principles rather than the Newtonian approach used in [4]. Although, both Newton's Laws and Lagrange's Equations are compatible, using Lagrange's Equations for multi degree of freedom systems with complex coordinate systems is said to more convenient [22], [23], [24]. Following up with the Lagrange's Equation for non-conservative forces,

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} + \frac{\partial R}{\partial \dot{q}} = \tau_q, \quad q \in \{\phi, \psi\} \quad (2.1)$$

Given that the Potential Energy and Kinetic Energy can be followed from Equations (2.2) and (2.3) respectively, below

$$V = g[(m_m l_m - m_t l_t) \sin \psi + 2m_{cw} l_{cw} (1 - \cos \psi)] + \frac{1}{2} k_\phi \phi^2 \quad (2.2)$$

$$T = \frac{1}{2} m_m \dot{r}_m^2 + \frac{1}{2} m_t \dot{r}_t^2 + \frac{1}{2} m_{cw} (\dot{r}_{cw,i}^2 + \dot{r}_{cw,j}^2) \quad (2.3)$$

The equations of motion for both degrees of freedom results as;

$$\ddot{\phi} = \frac{\tau_\phi + J \dot{\phi} \dot{\psi} \sin(2\psi) - k_\phi \phi - c_\phi \dot{\phi}}{J_\phi} \quad (2.4)$$

$$\ddot{\psi} = \frac{\tau_\psi - \frac{J \dot{\phi}^2}{2} \sin(2\psi) - c_\psi \dot{\psi} - g[(m_m l_m - m_t l_t) \cos \psi + 2m_{cw} l_{cw} \sin \psi]}{J_\psi} \quad (2.5)$$

For the description on definition of the moments and Simulink Model of Equations of Motion and the Nonlinear Model of the System, see Appendix A.

3. SLIDING MODE CONTROL

The main attribute of Sliding Mode Control is to make state trajectory reach the sliding surface within a finite time and maintain the trajectory in vicinity of the sliding surface which are known as the reachability condition for reaching phase and the sliding phase. Thus, these two phases have also been defined as $\sigma \neq 0$ for reaching phase and $\sigma = 0$ for the sliding phase. To be able to identify the reaching phase, an error function representing the difference between actual signal and the reference signal for both degrees of freedom is defined.

$$\begin{aligned} e_\phi &= \phi - \phi_d \\ e_\psi &= \psi - \psi_d \end{aligned} \quad (3.1)$$

Hence, by following [9] on the order reduction property of Sliding Mode Control, the Sliding Manifold will be given as,

$$s = \left(\frac{d}{dt} + \lambda \right)^{n-1} e \quad (3.2)$$

Where n represents the order of the system, λ is a strictly positive constant, and e is the error function as defined in the Equation (3.1). So, for a second order system of two degrees of freedom the definition of Sliding Manifold can be expanded into

$$\begin{aligned} s_\phi &= \left(\frac{d}{dt} + \lambda \right) e_\phi \\ s_\psi &= \left(\frac{d}{dt} + \lambda \right) e_\psi \end{aligned} \quad (3.3)$$

Therefore, the controller can be implemented as shown in the schematic below. (see Appendix B for Simulink representation)

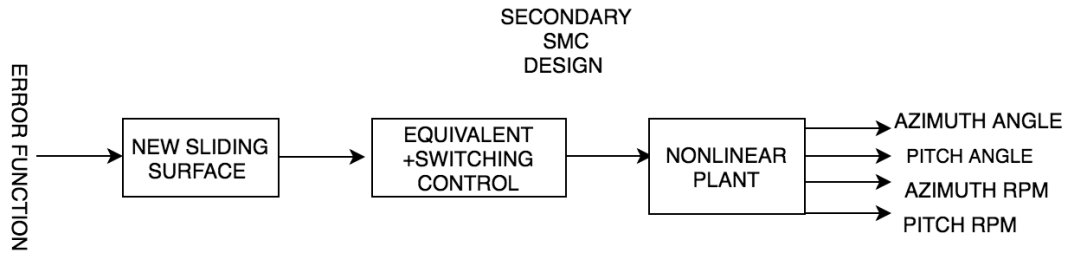


Figure 1 Representation of the Sliding Mode Control

4. LINEAR QUADRATIC REGULATOR

In order to create the Linear Quadratic Regulator, the first step should be recreating the system according to the given formation in Equation (4.1).

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \\ u &= -Kx \end{aligned} \tag{4.1}$$

To be able to reformulate the system into the state space structure above, the nonlinear system should be linearized. Due to the highly coupled nonlinear nature of TRAS, block by block linearization has been followed in MATLAB. Hence, the state space representation of TRAS is given as,

$$\begin{aligned} A &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2.2363 & 0 & -0.2982 & 0 \end{bmatrix} & B &= \begin{bmatrix} 0 & 0 \\ 8.0509 & 0 \\ 0 & 31.3047 \end{bmatrix} \\ C &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} & D &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned} \tag{4.2}$$

Following the check of stability of the system, controllability and observability of the states, by choosing sufficient Q and R matrices depending on the Cost Function and Algebraic Ricatti Equation, the gain matrix K can be calculated. Hence, the representation for the Linear Quadratic Regulator with the state vector x is given below. (see Appendix C for Simulink implementation)

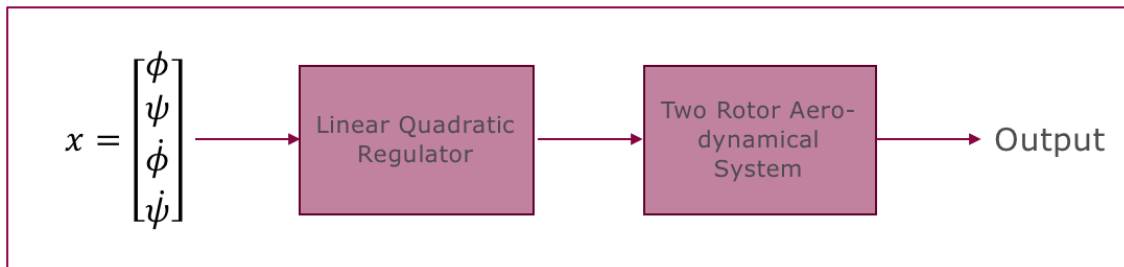


Figure 2 Representation of Linear Quadratic Regulator

5. RESULTS

5.1 PID Results

The Figure 3, below represents the tracking efficiency of the PID controller for both DOFs.

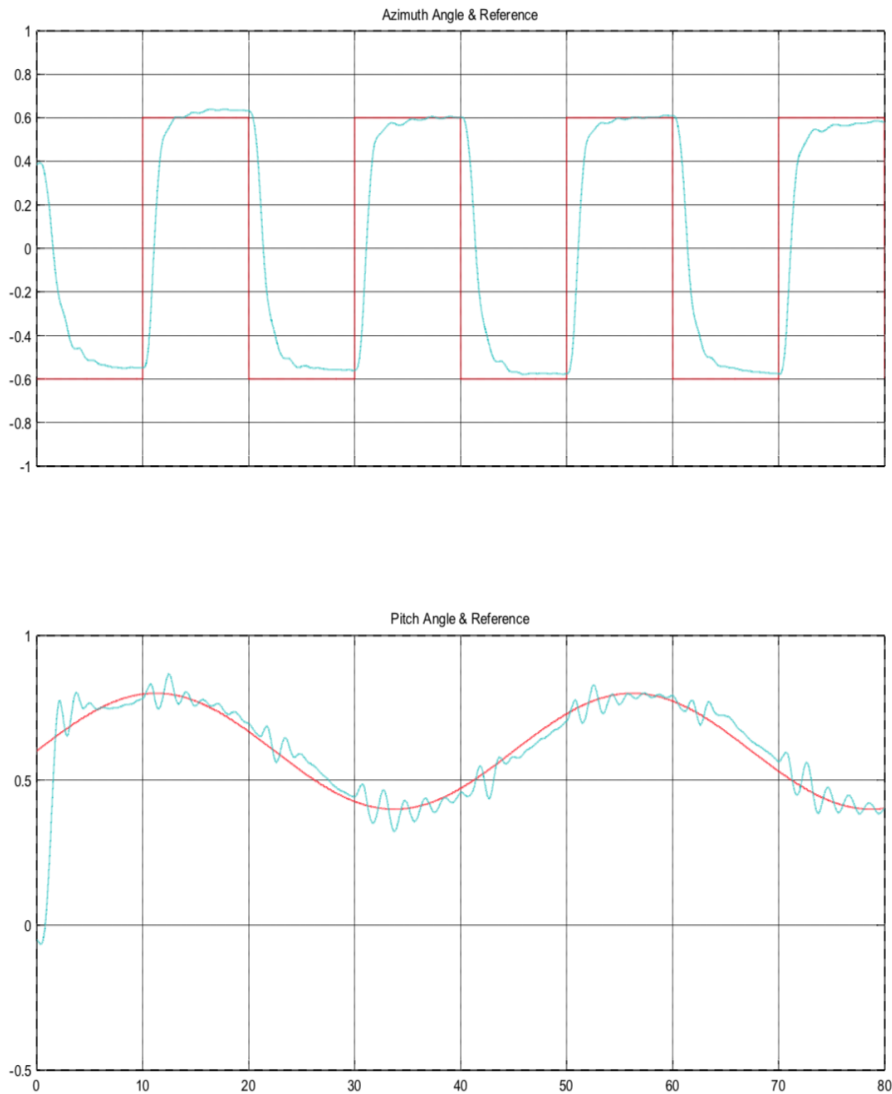


Figure 3 Azimuth and Pitch Angles and Its References for PID

Figure 4 below, shows the filtered signal, actuation of the control laws and trajectory tracking separately for both DOFs.

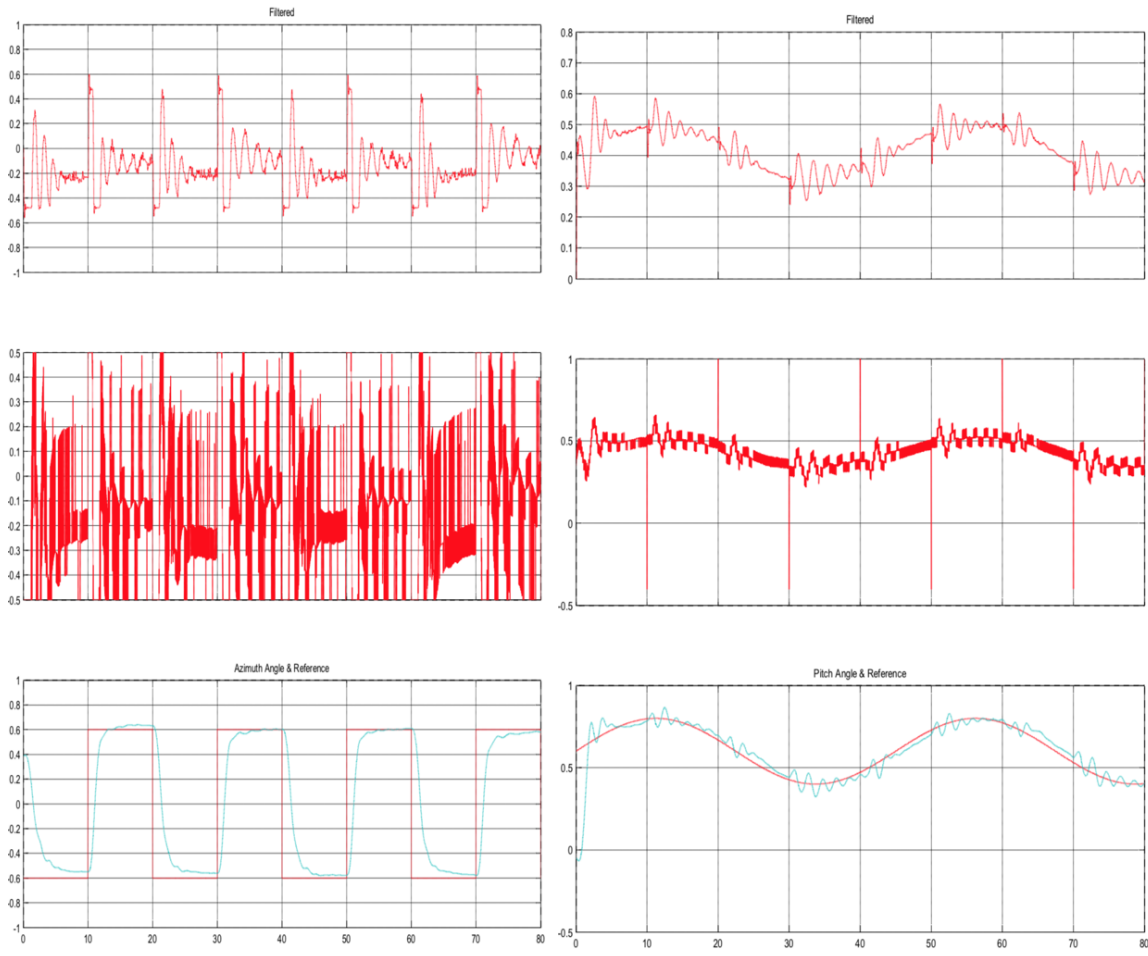


Figure 4 Filtered Signal and Response of PID for Azimuth and Pith

5.2 Sliding Mode Control Results

The Figure 5, below represents the tracking efficiency of the SMC controller for both DOFs.

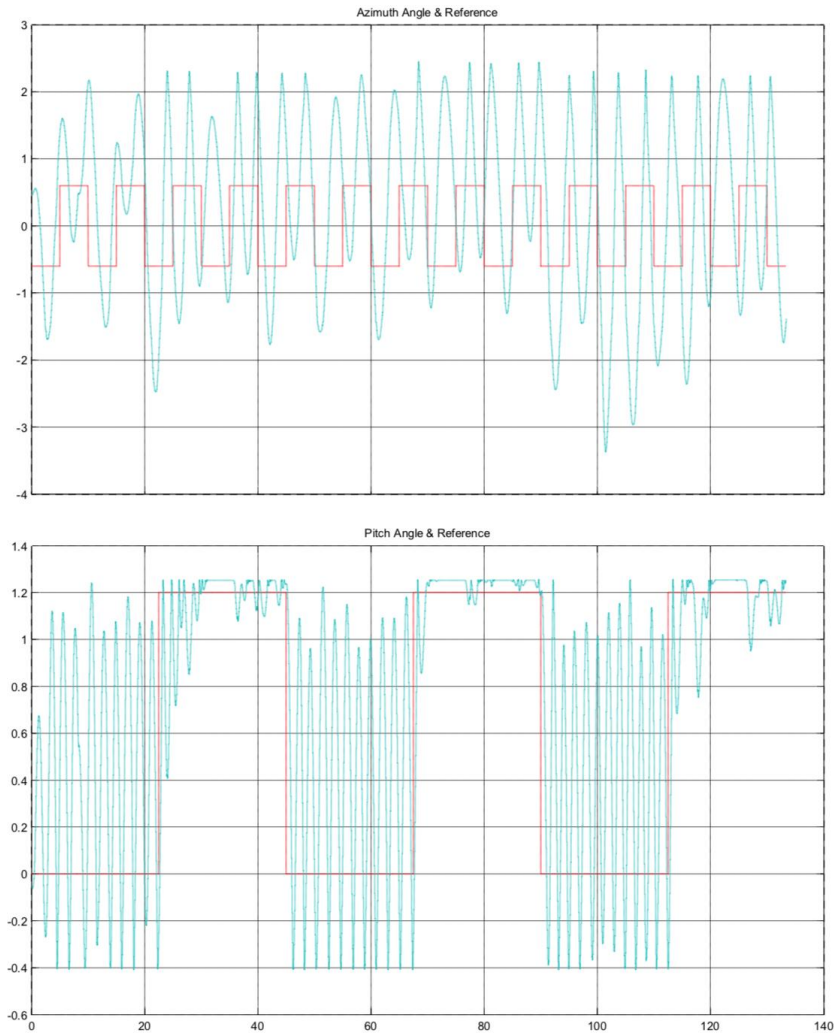


Figure 5 Azimuth and Pitch Angles and Its References for SMC

Figure 6 below, shows the filtered signal, actuation of the control laws and trajectory tracking separately for both DOFs. As it can be seen from the figure, the actuation time of the proposed SMC is impeccable. Yet, the tracking performance can be increased by reworking the constants in the system and introduce a filter that will effectively cuts out the noise in the reaction.

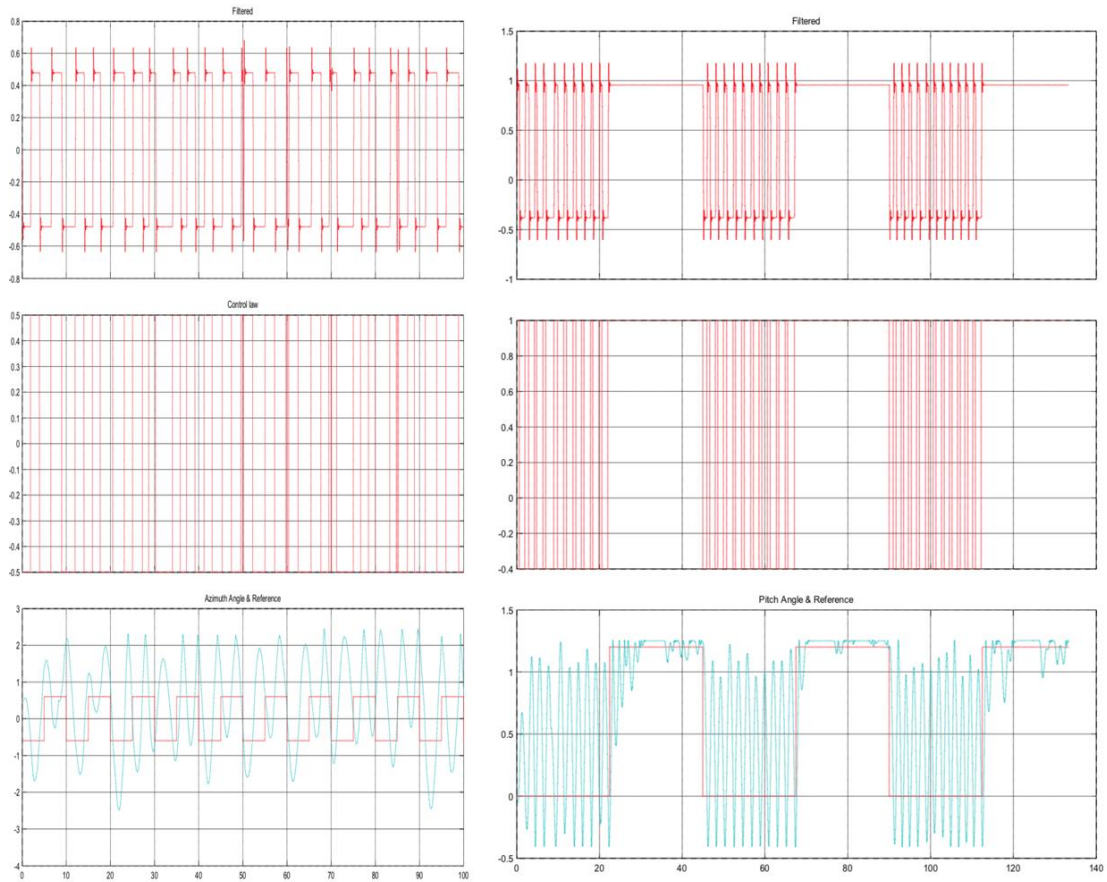


Figure 6 Filtered Signal and Response of SMC for Azimuth and Pitch

5.3 Linear Quadratic Regulator Results

The Figure 7, below represents the tracking efficiency of the LQR controller for both DOFs.

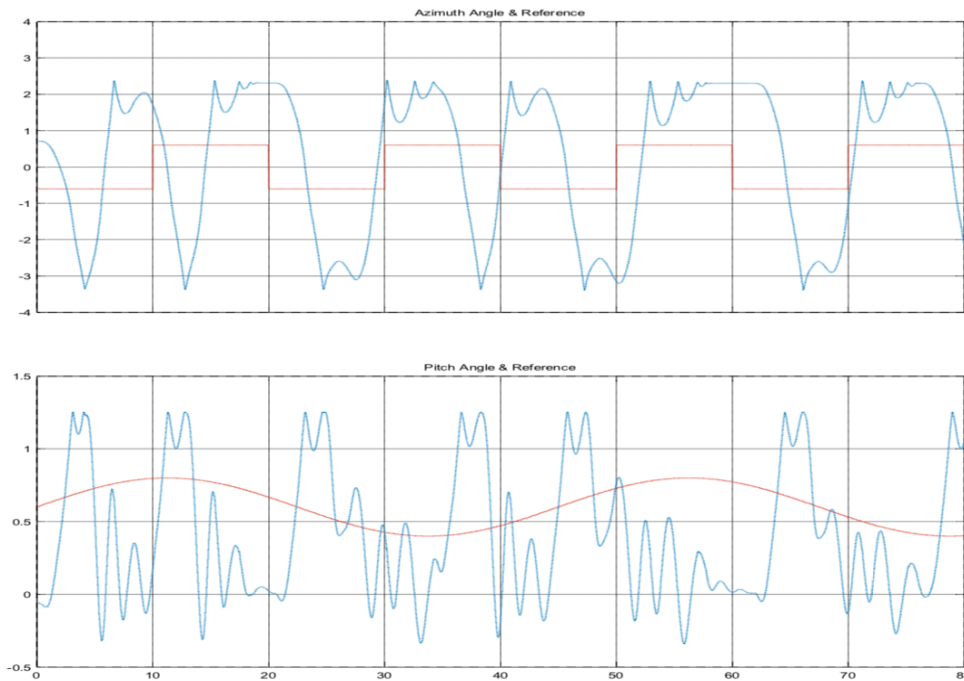


Figure 7 Azimuth and Pitch Angles and Its References for LQR

Figure 8 below, shows the filtered signal, actuation of the control laws and trajectory tracking separately for both DOFs. As it can be seen from the figure, the actuation time of the proposed LQR is impeccable. Yet, the tracking performance can be increased by reworking the constants in the system and introduce a filter that will effectively cuts out the noise in the reaction.

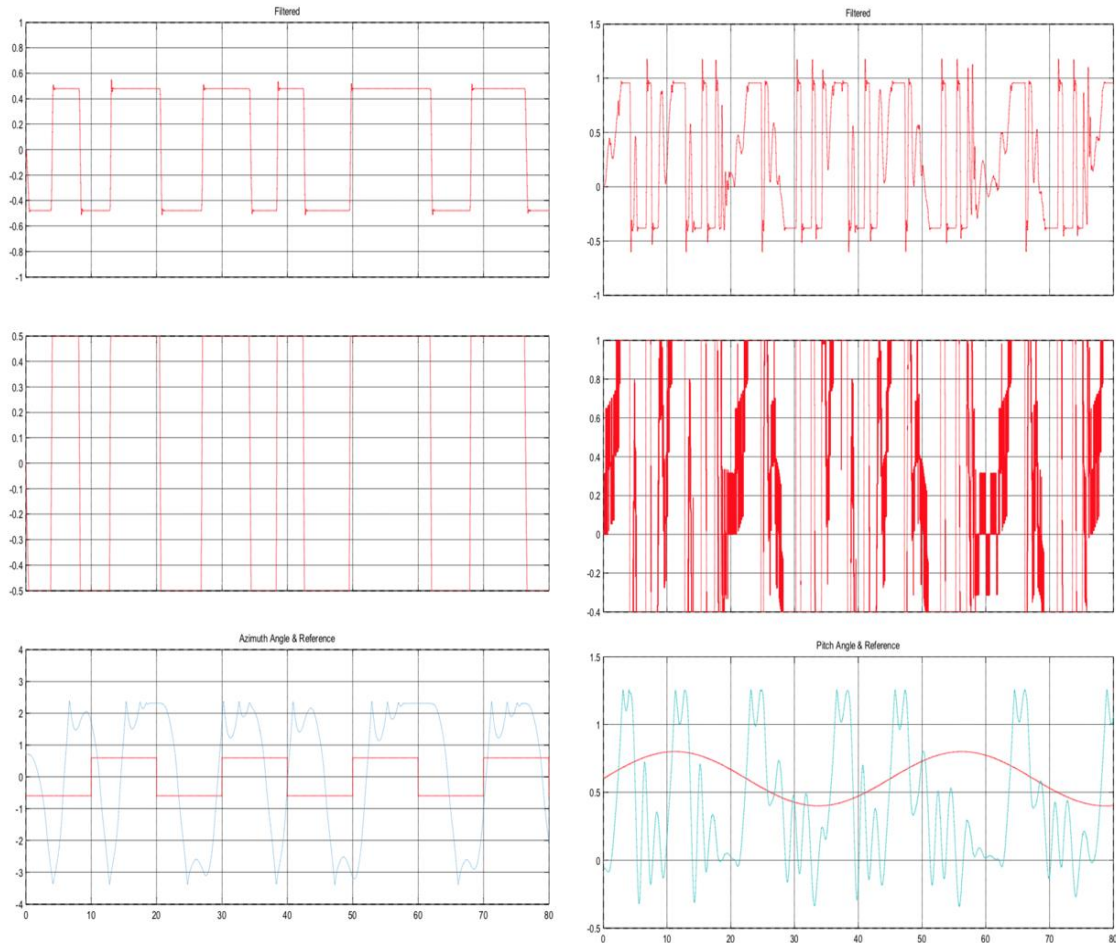


Figure 8 Filtered Signal and Response of LQR for Azimuth and Pitch

6. CONCLUSION

Throughout this research, the mathematical model for the highly coupled, nonlinear ‘Two Rotor Aero-dynamical System’ has been created, and different controllers have been implemented on the MATLAB’s Simulink environment. The results of the simulations have shown that both controllers have synchronous actuating moments and working at the perfect time but tracking performance can be increased by alternating the parameters.

When both controllers are taken into consideration there are some mutual advancements that could be proposed. To start with, Kalman Filter should be introduced into the system. From the literature research, it is seen that implementing the Kalman Filter cuts out the unwanted disturbances and creates a rather stable system. Whilst Kalman Filter will cut down the oscillations given in the tracking for the Sliding Mode Control, it will convert Linear Quadratic Regulator into Linear Quadratic Gaussian.

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CONFLICT OF INTERESTS

The author would like to confirm that there is no conflict of interests associated with this publication and there is no financial fund for this work that can affect the research outcomes.

REFERENCES

- [1] "Helicopter", *En.wikipedia.org*, 2018. [Online].
Available: <https://en.wikipedia.org/wiki/Helicopter>. [Accessed: 5-May-2018]
- [2] G. Padfield, "Helicopter Flight Dynamics". *Chichester: John Wiley & Sons*, 2008.
- [3] "Equipment | Alpha Control Lab", *A-lab.ee*, 2018. [Online].
Available: <https://a-lab.ee/equipment>. [Accessed: 1-May-2018]
- [4] Inteco, "Two Rotor Aero-dynamical System's User Manual", 2003.
- [5] K. U. Khan, N. Iqbal, "Modelling and Controller Design of Twin Rotor System/Helicopter Lab Process Development at PIEAS". [Online].
- [6] P. Chalupa, J. Prikyl, J. Novak, "Modelling of Twin Rotor MIMO System", *Procedia Engineering*, vol. 100, pp 249 - 58, 2015.
- [7] R. Rashad, A. El – Badawy, and A. Aboudonia, "Sliding Mode Disturbance - Observed Based Control of a Twin Rotor MIMO System", *ISA Transactions*, vol. 69, pp. 166 – 174, 2017.
- [8] G. Zhang and J. Furusho, "Speed control of two-inertia system by PI/PID control," *IEEE Transactions on Industrial Electronics*, vol. 47, no. 3, pp. 603-609, June 2000.
- [9] J. Slotine, W. Li, "Applied Nonlinear Control", Taipei: Pearson Education Taiwan, 2005.
- [10] W. Brogan, *Modern Control Theory. Englewood Cliffs: Prentice Hall, 1991.*
- [11] R. Bellman, R. Kalaba, *Quasilinearisation and Nonlinear Boundary – Value Problems*, New York: American Elsevier Pub. Co., 1965.
- [12] K. Young, V. Utkin, U. Ozguner, "A Control Engineer's Guide to Sliding Mode Control", *IEEE Transactions on Control Systems Technology*, vol. 7, no. 3, pp. 328 – 342, 1999.
- [13] Y. Shtessel, C. Edwards, L. Fridman, A. Levant, *Sliding Mode Control and Observation*, New York, NY: Springer New York, 2014.
- [14] V. Utkin, *Sliding Modes in Control and Optimization*, Berlin: Springer-Verlag, 1992.
- [15] C. Gurbani and V. Kumar, "Designing Robust Control by Sliding Mode Control Technique," *AEEE Research Publications*, vol. 3, no. 2, pp. 137-144, 2013.
- [16] R. M. Murray, Z. Li, S. S. Sastry, *A Mathematical Introduction to Robotic Manipulation*, CRC Press, 1993

- [17] U. Itkis, *Control systems of variable structure*, Toronto: John Wiley & Sons, New York, 1976
- [18] “Hyperbolic Tangent from Wolfram MathWorld“, Mathworld.wolfram.com, 2018. [Online]. Available: <http://mathworld.wolfram.com/HyperbolicTangent.html> [Accessed: 13-May-2018]
- [19] A. Janbakhsh, M. Bayani Khaknejad, R. Kazemi, “Simultaneous Vehicle Handling and Path Tracking Improvement Using Adaptive Dynamic Surface Control via a Steer-by-Wire System”, *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, vol. 227, no. 3, pp. 345 – 360, 2012.
- [20] “Graph of Logistic Curve”, Upload.wikimedia.org, 2018. [Online]. Available: <https://upload.wikimedia.org/wikipedia/commons/8/88/Logistic-curve.svg> [Accessed: 15-May-2018]
- [21] P. Lesniewski and A. Bartoszewicz, "Hyperbolic Tangent based Switching Reaching Law for Discrete Time Sliding Mode Control of Dynamical Systems," in *Recent Advances in Sliding Modes*, Istanbul, Turkey, April 2015.
- [22] S. Butt, H. Aschemann, “Multi – Variable Integral Sliding Mode Control of a Two Degrees of Freedom Helicopter”, *IFAC – PapersOnline*, vol. 48, no. 1, pp. 802 – 807, 2015.
- [23] S. Butt, R. Parabel, H. Aschemann, “Multi-Variable Flatness Based Control of a Helicopter with Two Degrees of Freedom”, *International Conference on Control Decision and Information Technologies*, pp. 321 – 326, 2014
- [24] S. Butt, H. Sun, H. Aschemann, “Control Design by Extended Linearization Techniques for a Two Degrees of Freedom Helicopter”, *IFAC – PapersOnline*, vol. 48, no. 11, pp. 22- 27, 2015.
- [25] O. Scholten, “Friction in Lagrange’s Formulation”, 2003. [Online]. Available: <https://www.kvi.nl/~scholten/AAM/Friction-Lagrange.pdf>, [Accessed: 2-May-2018]
- [26] C. -W. Tao, J. -S. Taur, Y. -H. Chang, C. -W. Chang, “A Novel Fuzzy-Sliding and Fuzz-Integral-Sliding Controller for the Twin-Rotor Multi-Input-Multi-Output System”, *IEEE Transactions on Fuzzy Systems*, vol. 18, no. 5, pp. 893 – 905, 2010.
- [27] S. Mondal, C. Mahanta, “Second Order Sliding Mode Controller for Twin Rotor MIMO System”, *Annual IEEE India Conference*, 2011
- [28] K. Vassiljeva, “Restricted Connectivity Neural Networks Based Identification for Control”, *TTU Press*, 2012
- [29] P. M. De Russo, *State Variables for Engineers*, John Wiley & Sons Inc, New York, 1965
- [30] “Signum Function”, Math.feld.cvut.cz, 2018. [Online]. Available: <http://math.feld.cvut.cz/mt/txtb/4/txe3ba4j.htm> [Accessed: 21-May-2018]
- [31] Wan Mansor Wan Muhamad, “Vehicle Steering Dynamic Calculation and Simulation”, *International Journal of Innovative Technology and Interdisciplinary Sciences*, vol. 1, no. 1, pp. 87-97, Nov. 2018.

[32] Setyamartana Parman, “Fuzzy Logic Control of Clutch for Hybrid Vehicle”, *International Journal of Innovative Technology and Interdisciplinary Sciences*, vol. 1, no. 1, pp. 78-86, Nov. 2018.

[33] John Pumwa, “Time Variant Predictive Control of Autonomous Vehicles”, *International Journal of Innovative Technology and Interdisciplinary Sciences*, vol. 1, no. 1, pp. 62-77, Nov. 2018.

[34] Nitin Afzulpurkar, “Automatic Control of Clutch Engagement and Slip for Hybrid Vehicle”, *International Journal of Innovative Technology and Interdisciplinary Sciences*, vol. 1, no. 1, pp. 49-61, Nov. 2018.

Appendix A

$$J = (m_m l_m^2 + m_t l_t^2) - 2m_{cw} l_{cw}^2$$

$$J_\phi = (m_m l_m^2 + m_t l_t^2) \cos^2 \psi + 2m_{cw} l_{cw}^2 \sin^2 \psi + J_z$$

$$J_\psi = (m_m l_m^2 + m_t l_t^2) + 2m_{cw} l_{cw}^2 + J_x$$

$$J_x = D \cos^2 \psi + E \sin^2 \psi + F$$

$$J_z = \sum_i^8 J_{iv} = 0.02421 \text{ kgm}^2$$

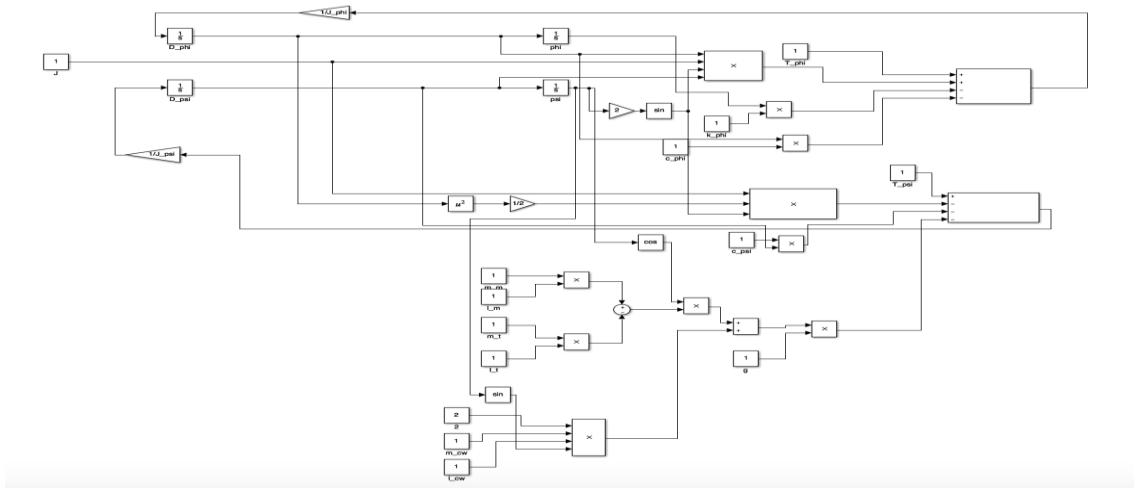


Figure 9 Simulink Model of the Equations of Motions

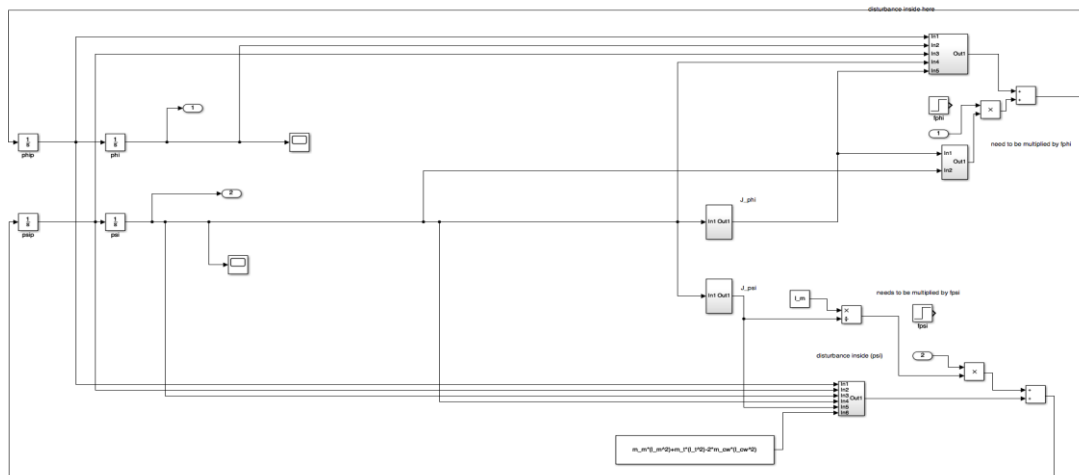


Figure 10 Simulink Model of the Nonlinear System

Appendix B

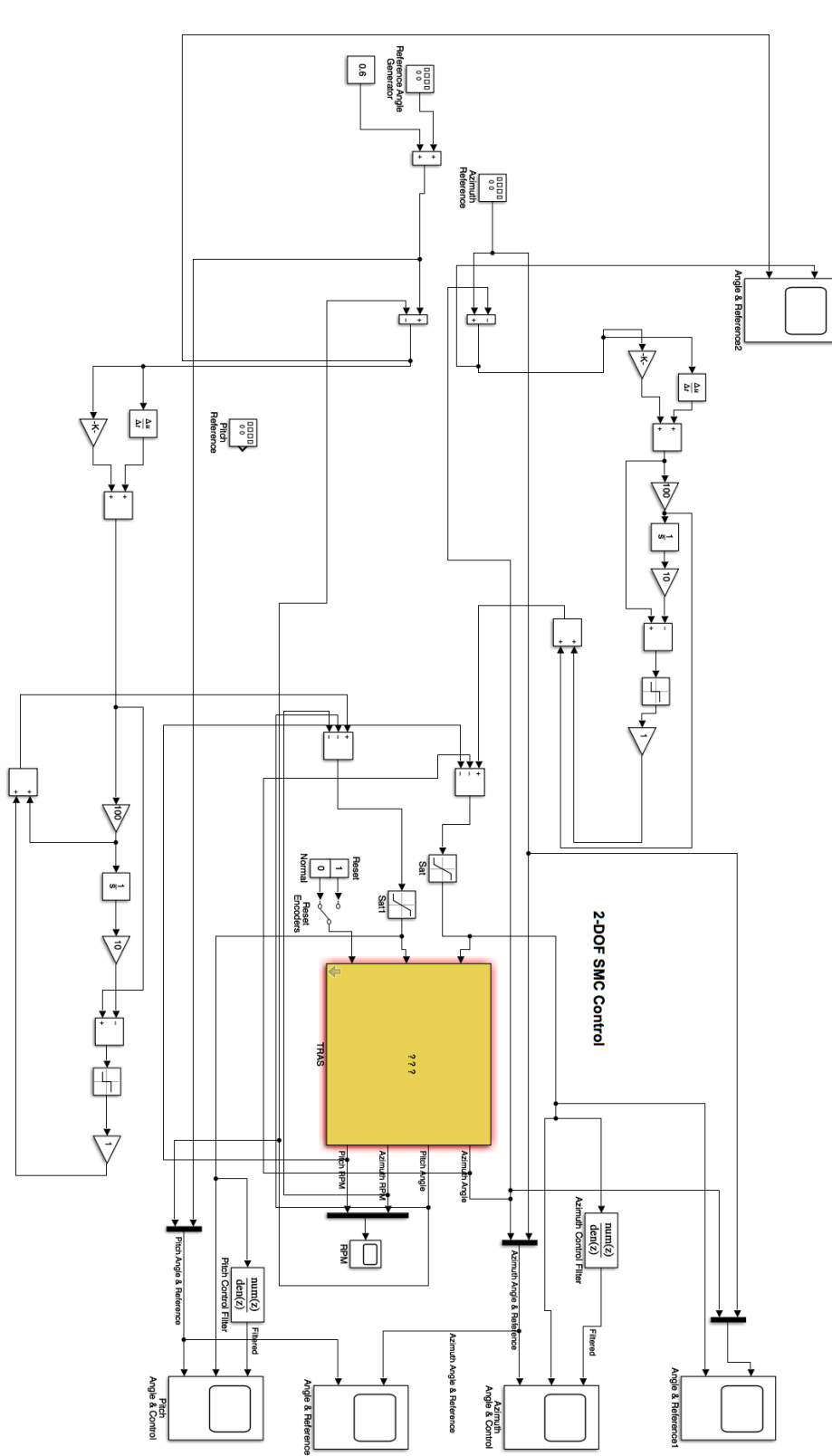


Figure 11 Simulink Representation of Sliding Mode Control

Appendix C

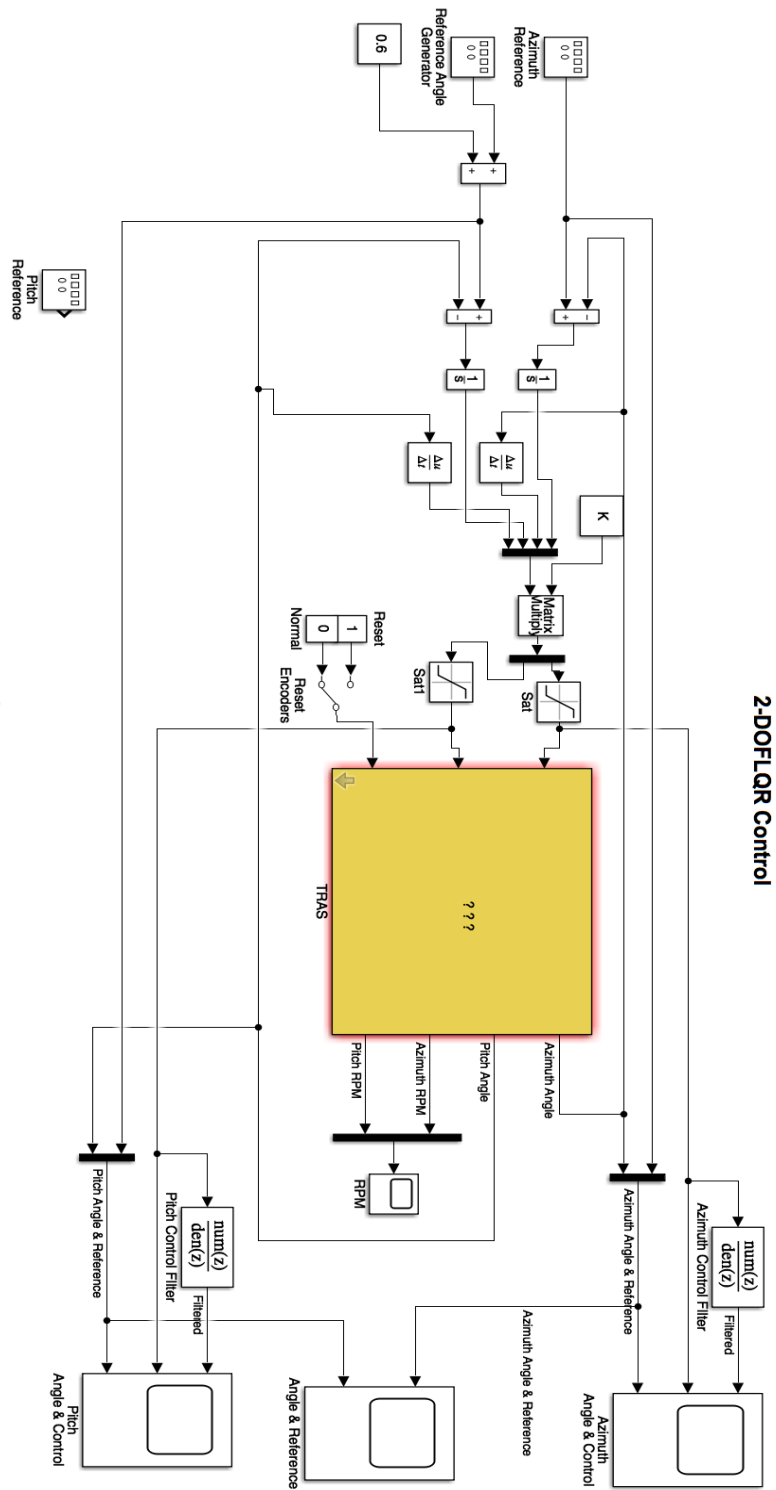


Figure 12 Simulink Implementation of Linear Quadratic Regulator