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An Evaluation of Dispersion Coefficient Models for Rivers

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Abstract
This article intends to evaluate a few mathematical and empirical models of river dispersion coefficients from previous studies. Two problems were cited as the causes of their shortcomings: the significant discrepancy between measured and predicted values of the phenomenon. The models based on previous research fail to take into account some of the geometric and hydraulic facts of dispersive flows, such as dead zones and bend effects, because they were made under assumptions that are false in real rivers. The empirical models omit some of the most significant parameters known to affect dispersion, whereas the mathematical models demand cumbersome, time-consuming, and labour-intensive tracer experiments. Although the accuracy of more recent machine learning techniques has increased, they are still very expensive, prone to error, and require a high level of expertise. All the equations fall short of the two crucial criteria for scientific acceptance: reproducibility and strong predictive power. A form for a new equation is proposed that will take into account many of the omitted parameters and, as a result, improve accuracy. Poor prediction accuracy should be addressed by the new equation. It is possible to derive the equation using dimensional analysis.

Keywords: *Assumption; Discrepancy; Dispersion; Irregularities; Models; Reproducibility*

INTRODUCTION

Pollution of rivers and streams occurs when pollutants are directly or indirectly introduced into them, and, depending on the degree of pollutant concentration, consequent negative environmental effects such as oxygen depletion and severe reduction in water quality may occur, affecting water quality [1], *and* [2], and possibly flora and fauna. Fortunately, this water body poses some characteristics by which they are able, to some degree, contain the usually excessive pollution load and eventually recover their purity. The most important of these characteristics is the rapid mixing and dispersive ability by which it spreads out and dilutes a mass of pollutants. This characteristic is measured by a term called dispersion coefficient or its dimensionless equivalent called dispersion number (D = UdL, where D is dispersion coefficient; U, longitudinal velocity; d, dispersion number; and L, length of channel).

Beyond a certain distance along the river stretch, the longitudinal flux of the contaminant can be expressed as proportional to flux of the stream-wise concentration gradient. The proportionality coefficient is the longitudinal dispersion coefficient D [3], an arbitrary nonlinear variable that fluctuates widely, used in the measurement of the dispersive ability of the regime. Reliable estimation of D is important for water pollution control and design of treatment plants [4], [5], and [6]. It is determined by the usually expensive, time consuming and laborious measurements of tracer concentration against time values [7], [8],[9], and [1]. Several research have produced several models in attempt to eliminate the cumbersomeness in determination and disparity between measured and predicted values of D. Some of the models are mathematical while others are empirical and more recently, algorithmic. There is still however, no universally accepted model owing to large disparities between measured and predicted values of D [10], [11], and [12]. These disparities suggest that the processes contributing to dispersion are not yet well understood [12]. This paper presents a critical review of some of the models available in literature.

LITERATURE REVIEW

Taylor in 1954, introduced the concept of dispersion coefficient in the one-dimensional diffusion equation [13]. He established that in a long, straight pipe with laminar flow, the diffusion and convective processes occurring through-out the cross section interact to produce longitudinal dispersion coefficient. Taylor's equation was [14], [15], [2],[16], [18].

$$A\frac{\sigma c}{\sigma t} + AU\frac{\sigma c}{\sigma t} = \frac{\sigma c}{\sigma x}\left[DA\frac{\sigma c}{\sigma t}\right] \quad (1)$$

Where A is cross-sectional area of the stream; C is cross-sectional average concentration of the pollutant; U is mean longitudinal flow velocity, K is longitudinal dispersion coefficient (or D) at a distance x from point of introduction of pollutant and t is time of concentration at point x.

Many investigators have subsequently applied Taylor's concept to flows in natural open channels, not with-standing Taylor's own recognition that his model applied specifically to flows in long straight pipes. This implies that application of the model to natural channels with all their geometric and hydraulic irregularities cannot produce true and acceptable results of D. All other equations emanating from Taylor's line of thought follow the same concept and assumptions and will expectedly be distressed with the same limitations as with assumptions in Taylor's equation.

The main assumptions in Taylor's equation are that the flow is homogenous and isotropic, and the channel or river section is prismatic, so the flow is uniform. However, the sinuosity of bends and the consequent centrifugal forces generated in them affect both geometric and flow uniformity. Consequently, any model based on Taylor's analysis has a clear weakness in determining the actual longitudinal dispersion coefficient in natural water courses that have bends.

The study in [3] suggested that Taylor's analysis does not correctly describe the entire process of dispersion. Two periods, the convective period in which the movement of tracer particles is influenced by their initial convective velocity forming a longitudinally skewed cloud which does not follow the Taylor one-dimension diffusion equation, and a later diffusion period during which Taylor's analysis applied and the initial distribution decays according to the diffusion equation [18], [7]. Only in the later period is it possible to speak of a dispersion coefficient which correctly describes the process [3]. The solution of equation (I) is [13]

$$C = \frac{Mo}{A\sqrt{4\pi Dt}}\exp\left[-\frac{(x-)u^2)}{4Dt}\right] \quad (2)$$

M is mass of pollutant introduced into a stable river; C is cross sectional average concentration of pollutant. It is the determination of D using equation (2) that makes tracer studies expensive and time consuming, because samples must be taken down stream of the reach with the associated model calibration and verification [13].

The moment method is based on evaluating the variance (σ^2) of the solution of the one-dimensional dispersion equation for a non-settleable tracer as given in equation 2. The method depends on the second moment which tends to magnify the long tail observed in channels which does not give accurate evaluation of D [10], [7]. Thackson observed that the method does not provide any logical procedure for the estimation of the flow and settling velocities. The unavoidable natural occurrence of dead zones that establishes long tail and skew observed in the concentration-time data plots should considerably affect this method because of the assumptions in its formulation. Moreover, it is known that flow discontinuities that result in dead zones significantly raise the value of D[10]. The generalized moment as an analysis tool method may be useful in analyzing transvers mixing data, but obtaining accurate measurement of the net transverse velocity is difficult and produces large errors.

Another major flaw of equation (1) is Taylor's assumption that shear velocity and transverse mixing are in equilibrium after a certain time scale at some point downstream. It is extremely difficult to see how this can occur in a natural stream given all its irregularities, hydrodynamics, and geometric variations. This can only be imaginary, and if it occurs at all, it must be an instantaneous localized event of unnoticeably short duration. Equilibrium cannot be reached in meandering flows because of the effect of centrifugal forces generated at the bends, nor is it possible even in straight channels where there is frictional resistance that produces continuous shear.

Taylor's analysis of solute spread by the joint action of turbulence and sheer velocity profile in circular pipes resulted in equation (3) [19], [20], [21].

$$D = 10.1rU \tag{3}$$

Where K is dispersion Coefficient, r is radius of pipe and U is sheer velocity.

Taylor's basic line of thought provided springboard for many other researchers to proffer somewhat better models for D. [22] relying on Taylor's model assumed a logarithmic velocity profile for channel flow of infinite width (eliminating transverse velocity gradient) proposed a longitudinal dispersion coefficient with Von Karman constant (K) of 0.41 and added the depth averaged value for equation (3) ($\bar{\varepsilon} = 0.067dU_*$) as follows.

$$D = (5.86 + 0.067)dU_* = 5.93dU_*$$

$$D = 5.93dU_A \tag{4}$$

Where d is depth of flow and $\bar{\varepsilon}$ is depth averaged vertical diffusion coefficient. Both equations (3) and (4) do not contain the (w/h) and $\left(\frac{U}{U_*}\right)$ terms called aspect ratio and friction factor respectively, adjudged to be the two most important terms in the determination of D [23]. Aspect ratio is the consequence of transverse difference of longitudinal velocity [24], [20], [25], [6], [26] . Equation (4) assumes vertical velocity gradient to be the most important factor affecting dispersion in contrast to longitudinal velocity. It should be noted that vertical velocity has very short duration occurring at the point of pollutant introduction while longitudinal velocity has long duration and should be considered the most important.

Elder used small laboratory flume to verify his formula. However, later laboratory works have shown much higher results for D. In [18], has been reported that values of D obtained in natural streams have ranged from 40 to 800 DU_*, the average being $300DU_*$. The large variations associated with Elder's equation is mainly the result of ignoring the effect of distribution of lateral velocity [6] and has cast doubts on the applicability of Taylor's analysis to open channel flows [3].

In research [27] stated that Elder's equation for longitudinal dispersion in 2-dimensional solute transport analysis is not applicable to meandering channels because the vertical distribution of the longitudinal velocity does not obey the logarithmic law in the bends of an open channel. Elder's equation in 2-dimension for an infinitely wide-open channel (assuming the vertical distribution of longitudinal velocity as the logarithmic function proposed by Von Karman) is

$$u - \bar{u} = \frac{U_*}{D}(1 + \ln y') \qquad (5)$$

u *is* longitudinal velocity; $\bar{u}$ *is* vertically arranged velocity; U_* *is* frictional or shear velocity; k is Von Karman constant; y' *is* dimensionless vertical coordinate defined as y/d and d water depth. The vertical diffusion coefficient in Fischer's popular triple integral equation reproduced as equation (9) is given as

$$\varepsilon = DU_*(1 - y') \qquad (6)$$

By substituting equations (5) and (6) into equation (9), the triple integral results is

$$D = \frac{0.404}{K^3} dU_* \qquad (7)$$

This equation (7) has a theoretical background and is expressed using simple constants; hence it has been widely used to determine longitudinal dispersion coefficient in two-dimensional solute transport analysis. [28] added a sine function to a power law to obtain.

$$u - \bar{u} = \frac{AU_*}{K}(y' - 0.1)^{0.5} + Bsin^2 \pi y' \qquad (8)$$

A and B are the regression coefficients determined experimentally. Equation (8) can be used for bends, and when B is set to zero, it can be used for straight channels. Equation (8), used either for meandering or straight channels, still demands the rigorous determination of experimental data. Though the method of derivation of equation (8) is not known, it can be seen that the coefficients A and B must be determined through the usually rigorous experimental procedures. This makes it not a ready tool for the engineer.

In [3] it has been used lateral distribution of averaged depth velocity instead of vertical profile as considered by Elder to obtain,

$$D = \frac{1}{A}\int_0^B h(y)U'(y) \int_0^y \frac{1}{\varepsilon y h(y)} \int_0^y h\,(y)U'^{(y)} dydydy \qquad (9)$$

Where A is cross sectional area; B is channel width; h(y) is lead water depth; y is coordinate in lateral directions, e is local transverse turbulent (mixing) coefficient and U(y) is deviation of local depth mean velocity from cross sectional water velocity.

The fundamental difficulty in using equation (9) is that it requires elaborate transverse velocity profile and cross-sectional geometry which are not readily available [12]. The equation does not consider vertical velocity profile which have been shown to play significant role in pollutant spread [25], [29], [30]. As is common with mathematical models, equation (9) requires large data computations or formulation which may lead to significant errors resulting from attention loss. No researcher has confirmed the appropriateness or reliability of the equation. It does not also take into consideration the many irregularities that affect dispersion in rivers such as sinuosity effect, neither can it be an on-site assessment tool for the engineer.

Following the many inadequacies of equation (9), [24] introduced reasonable approximations in the triple integral velocity deviations and transverse dispersion coefficient to arrive at an empirical formula.

$$D = \frac{0.11U2B}{HU*} \quad (10)$$

Equation (10) initially had a wide acceptance for its simplicity and theoretical background, but large variations between predicted and measured values has reversed positive opinions on it. The variations resulting from this equation are thought to be owing to the fact that no stream completely fulfils the assumptions inherent in the development of it [12].
In the line of mathematical models of longitudinal dispersion coefficients available in literature is the more recently developed model by Aguwamba and verified by Uneke [11] given as

$$\tau^2 = Ut\left(\frac{Dt}{\pi}\right)^{1/2} exp\left(-\frac{U2t}{4D}\right) + \frac{1}{2}(U2t2 + 2Dt)erfc - \left[\frac{\frac{u}{2}\left(\frac{t}{d}\right)1}{2}\right] - \left[\frac{Ut}{2}erfc\left(-\frac{U}{2}\sqrt{\frac{t}{D}}\right) + \sqrt{\frac{Dt}{\pi}exp - \frac{U2t}{4D}}\right]^2 \quad (11)$$

$$\tau = variance = \frac{\sum Cx2}{\sum C} - \left(\frac{\sum Cx}{\sum C}\right)^2 \quad (12)$$

Equation (11) is based on integration of modified equation (2) for a constant time, variable distance sampling method. One difficulty in equation (11) is its usage without a computer. Also, it requires for its use the process of sampling and laboratory analysis as does Taylor's equation. As can be seen, it requires large data handling thus making its usage prone to mistakes. As with other mathematical equations it does not recognize geometric irregularities which are known to affect dispersion coefficient. Though the theory behind the equation reduces tracer time, but its prediction of the rising and falling limb of the characteristic bell-shaped figure associated with tracer studies, are deviated [11].
[25] by direct integration of Fischer's equation (9) and with some other existing equations involving depth variation and lateral distribution of the deviation in velocity, derived.

$$\frac{D}{HU*} = \frac{0.15}{8\varepsilon_r}\left(\frac{B}{H}\right)^{5/3}\left(\frac{U}{U*}\right)^2 \quad (13)$$

ε_r is transverse mixing coefficient.

The basic assumption for the development of equation (13) limits its used to only straight uniform rivers [25]. Based on Fischer's triple integral equation (9). Owing to the assumption that in principle limits the use of equation (13) to straight uniform rivers, Deng et al attributed the shortfall between measured and predicted values to dead zones, bends and other irregularities despite the use of a coefficient of 0.15 determined from what they called comprehensive revision constant (y) and which should have accounted for these factors.
Although they claim superiority of their equation over that of Seo and Cheong because of improved transverse mixing coefficient, the coefficient of determination of their model is 0.21 while that of Seo and Cheong is 0.4 [31]. This puts in doubts other numerical superiority claims of the equation. Using a set of data provided in [12] only about 12.5% of the predictions by this equation was good.

The study [6] employed triple integral similar to Fischer's and regression analysis, assuming a non-linear relationship between laboratory and natural channel to obtain.

$$\text{In}\left(\frac{D}{HU*}\right) = \Psi\text{In}\left(\frac{K2}{HU*}\right) + \Psi_{0,} \quad (14)$$

in which Ψ and Ψ_0 are regression coefficients, $D_1$and D_2 are dispersion coefficients for laboratory and natural channels respectively. The final equation after regression is

$$\frac{D}{HU*} = 17.648\left(\frac{B}{H}\right)^{0.3619}\left(\frac{U}{U*}\right)^{1.16} \quad (15)$$

Equation (15) identifies with straight channels only. The accuracy of the model represents only the result of the data used in its development. Not less than 80% of the training data was from the field data and 20% was used for verification. The claim of better **MER** and **MAER** is based on few selected data from the same field data used in its development. A look at table 1 of measured and predicted values by equation (15), shows that the predicted values are very far from the measured in all 16 data sets used. The accuracy of this equation depends on the accuracy of transverse mixing coefficient. Also, the data sets used in its development and verification did not include irregularities that naturally occur in rivers and canals.

Beyond Taylor, Elder and Fischer, the use of mathematical models to address the issues of longitudinal dispersion in natural channels is on the increase showing their roles as scientific tools to improve the understanding of the mechanism. However, the process has not improved the accuracy in determination of the phenomenon. The above equations represent some of the initial developments in the studies of dispersion and self-purification capacities of water bodies as waste disposal systems. The more recent models are largely empirical, albeit with application of dimensional and regression analysis; some of them also starting from the one-dimensional diffusion equation and some others relying on Fischer's triple integral equation.

Hydraulic and geometric properties were first introduced into dispersion by [32] as they combined one-dimensional and dispersions equations to arrive at a model for longitudinal dispersion coefficient for Froude number less than 0.5.

$$D = \frac{0.058HU}{5}; \quad F_n<0.5 \quad (16)$$

Where S is slope of energy grade line and F_n is Fraude number from numerical models but they are not adequate representations of the factors that affect dispersion, hence the unpopularity of the equation. The equation does not include the aspect ratio (B/H) and the friction (bed material roughness) $\langle\frac{U}{U*}\rangle$ terms, which are very important in describing dispersion mechanism [34], [33], [30], [24], [23], [21], [19].

[32] used similarity between the linear one-dimensional flow equation and the dispersion equation to develop equation (16). [24] argued that the similarity was improper because the mechanism of dispersion of a flood wave and a solute are quite different.

[35] starting from Fischer's equation (9) and taking into consideration the effect of lateral velocity gradient wrote

$$D = \propto \frac{\boldsymbol{U2B2}}{\boldsymbol{U*H}} \quad (17)$$

Where $\propto = 0.18\left(\frac{U}{U*}\right)^{1.5}$

Equation (12) underestimates [25], and [6] despite the parameter ∝ being of channel geometry and cross-sectional velocity gradient. [35] suggested that the parameter ∝ can be determined by considering sinuosity, sudden contractions and expansions and dead zones. These are very difficult parameters to determine, some of them being in the abstract.

[21] applied dimensional and regression analysis [36] on the one-step Hubber method (a non-linear multi regression method) to obtain.

$$D = 5.915\left(\frac{B}{H}\right)^{0.62}\left(\frac{U}{U*}\right)^{1.428}(HU) \quad (18)$$

They stated that their equation is superior to others preceding it. Comparing their result with others, they reported a coefficient of correlation 0.75, but [31] reported a coefficient of correlation of 0.634 for the same equation with the same data sets. Although sinuosity and bed factor were initially included in the development of equation (18) they were later expunged for lack of data, the authors still recognizing that these factors affect dispersion [21].

[37] used the original theory of Fischer and applying the Von-Kamman defect law, derived equation of the form

$$D = 0.6U_*\frac{B^2}{H} \quad (19)$$

Though equation (19) incorporates aspect ratio [B/H), it fails in the friction term and other geometric irregularities. Of the eight models compared by [6], equation (19) has the largest **MER** and **MEAR** showing that it overestimated D far beyond the measured.

[38], on the basis of the data obtained from Deng et al. predicted longitudinal dispersion coefficient by artificial neural network, with no defined equation as model. Their method is full of uncommon terminologies and long processes that do not permit for on-the-spot assessment of dispersion coefficient. Their report of R^2 value in training and 0.69 in testing indicate that the result is of high inaccuracy. It can be concluded that the model overestimates D [31].

The study in [20] used dimensional and regression analysis on 81 data sets to obtain.

$$D = 10.62(HU)\left(\frac{U}{U*}\right) \quad (20)$$

Equation (20) does not have the important parameter B/H and other factors that estimate the effects of bends commonly found in natural rivers. This may be responsible for the coefficient of correlation (R^2) of 0.84 obtained for the equation. Combining equation (20) and Seo and Cheong equation (18) and by trial and error, they obtained.

$$D = \left[7.428 + 1.775\left(\frac{B}{H}\right)0.62\left(\frac{U}{U*}\right)0.572\right]\left(\frac{U}{U*}\right)HU \quad (21)$$

They criticized [24], [37] equations as over estimating D. [39] reported that there are errors in research methods that led to the derivation of equation (21). This is in addition to the claims by Noori et al of mistakes in results presented.

[12] incorporated the effect of sinuosity, an important parameter for river transverse irregularity and gave D as.

$$\frac{D}{HU*} = 2\left(\frac{B}{H}\right)^{0.75}\left(\frac{U}{U*}\right)^{1.37}S_i^{1.52} \quad (22)$$

Using root mean square error (**RMSE**) as a performance index equation (22) is behind others for D >100 and W/H> 50. Equation (22) is inadequate for forecasting D in sinuous rivers [12] as it does not recognize those parameters in its development.
[40] also used dimensional analysis to obtain.

$$D = 5.4\left(\frac{B}{H}\right)^{0.7}\left(\frac{U}{U*}\right)^{0.13}HU \tag{23}$$

They admitted that their equation was good at predicting D values in trapezoidal flumes but overestimated it in rectangular flume.
[23] applied geometric algorithm to 65 data sets to obtain.

$$\frac{D}{HU*} = 2\left(\frac{B}{H}\right)^{0.96}\left(\frac{U}{U*}\right)^{1.25} \tag{24}$$

They confirmed that the most important parameter for accurate prediction of D is the ($\frac{U}{U*}$) term. They claimed that models by [21], equation (18) and [25] equation (13) performed well in estimating D only the values are less than $100m^2/s$. Equation (19) does not recognize the channel geometric properties for Sinuous rivers and shows poor predictive performance in table 1 and even when RMSE and percentage errors are used [12].
All empirical models are obtained with regression analysis and are a product of precise data and as such, the models can actively perform well when all conditions governing models generation are met. However, these models soon prove inefficient owing to changes in river configurations resulting from climate change. The models remain valid only if the rivers are constantly re-measured for model recalibration. This way model accuracy is preserved [1].
[27] proposed a model by slightly modifying Mozafari's equation (8) and obtained.

$$D = \frac{d^2}{\varepsilon}\left\{-0.0258\left(a - 0.38\frac{U_*}{K}\right)^2 + 0.0778\left(\frac{U_*}{K}\right)^2\right\} \tag{25}$$

At $a = 0.38\frac{U_*}{K}$ the maximum value of D is obtained as

$$D_{max} = 0.0778\left(\frac{d^2}{\varepsilon}\right)\left(\frac{U_*}{K}\right)^2 \tag{26}$$

At $a = 0$, the equation (26) can be used for straight channels.

Whereas Elder used vertically distributed diffusion coefficient, [27] used $\bar{\varepsilon}$ as an average value $(i.e\ \bar{\varepsilon} = 0.067dU_*)$ Setting a at zero, equation (26) becomes.

$$D = 0.071\left(\frac{d^2}{\bar{\varepsilon}}\right)\left(\frac{U_*}{K}\right)^2 \tag{27}$$

[31] affirmed that the accuracy of D estimation could be improved by accounting for curvature and that aspect ratio showed greater input on D than friction factor. Some of the most recent empirical models include those of [41], [6].

D/(HU*) = 2.827 (W/H) 3.7613 (U/(U*))1.4713 (28)

Wang and Huai, [6]

D/(HU*) = 17.648 (W/H) 0.3619 (U/(U*))1.16 (29)

They used average value of $\bar{\varepsilon} = 0.0.67dU_*$ for ease of calculation. Setting Von Karman constant as 0.434 instead of 0.41, then the two equations (4) and (27) are equal. Elder's equation has been found not to produce good results of D, same would apply to equation (27)

These empirical methods are increasingly showing dependable results [42], [43], [44], [33] used data set from 29 rivers to present an empirical predictive equation for longitudinal dispersion coefficient. They concluded that their model was better than previous empirical models and that Fraude number was important in integrating the slope of the channel. [30] established an empirical equation that involved the width, depth, cross-sectionally averaged velocity and bed shear velocity of 116 data sets. They claim a superior model to others. [45] also 128 data sets from 41 natural rivers in the USA to obtain a model thought to be more reliable than other predictive models.

MATERIALS AND METHOD

The materials used in this work include literature from world acclaimed authorities in dispersion studies. These materials revealed the methods and techniques used in development of the models including the assumptions and theories behind them. The short-comings, errors, weaknesses and strengths of the models were obtained in this way.

The method of authentication of the claims of the models was by adopting tables and figures in literature and applying statistical and regression analysis to establish the veracity or otherwise of some of the models as claimed by the authors.

Machine Learning Techniques

The AI algorithm is used in almost every aspect of human endeavour including psychological science, notwithstanding the ethical concerns [46]. In recent times, Artificial Intelligence (AI) and Machine learning techniques are being used to predict D. The machine learning techniques include Model Trees (MT), Artificial Neural Networks (ANNs) and Support Vector Machine (SVM). These methods are intended to remove the disadvantages of regression- based method. The accuracy of genetic program (GP) expression implemented by [12] indicates that the GP models are better than empirical models in predicting dispersion coefficient. Sinuosity was considered to be a critical input variable for D. FFA hybridized with ANFIS model is used to improve the accuracy of estimations including the roller length of hydraulic jump and monthly stream flow forecasting [13].

[15] used various machine learning algorithms including GPR, SVR, M5P and RV to estimate DL. They found that M5P gave the best result. They used M5P to formulate the model in equation (30) and (31). They claim that the M5P models are better than other models from other machine learning and empirical models. The main advantage of M5P models is their ability to provide practical mathematical formulae as in equations (30) and (31) which are highly applicable to DL estimations.

$$D/(HU^*) = 1.6896(W/H) + 20.0124(U/(U^*)) + 393.3343 \quad (30)$$

$$D/(HU^*) = 2.8759\,(W/H) + 181.7915(U/(U^*)) + 339.5557 \quad (31)$$

Machine learning artificial intelligence is emerging as a leading and more accurate method of modeling in the engineering field. Several researchers in the field of environmental engineering [47], [42], [40], [17], [4] and particularly on the dispersion coefficient topic have proposed models involving the use of AI models which come in many terms as ANN (based on the different learning algorithms (Radial Bases Function Neural Network, (RBFNN) feedforward backpropagation neural network (FBNN), and the generation regression neural network), [8]. Al models show better predictions of the D than the empirical models, and prediction accuracy seem to grow as new improved variants of the machine learning techniques emerge. Other evolution machine learning tools include genetic programming as used by [12] and [48]. The M5 model tree [49], gene expression programing (GEP) used by [34]; support vector regression (SVR), [31] etc, all show progressive improvement in computational speed and accuracy of D prediction and thus judged better than empirical models.

Hybrid versions of the AI models involve the combination of two or more models for improvement of the learning process and outcome, [50] and has shown better results than those single AIs and the empirical models, [8]. and [51] investigated the efficiency of Three Bat-Inspired algorithm optimized intelligent models including Optimized Neural Network ONN, Optimized Fuzzy Inferrence System (OFIS) and optimized support vector regression (SVR) and their combinations in estimating D. The optimization eliminated the associated loss in accuracy of the intelligence models, thereby improving accuracy of model.

In recent times, about 67% of prediction of D has been by AI and with 39% the formulae involved being by AI techniques. About 33% of predictions have been by empirical methods [2]. Generally, 30% of D predictions are by equation-based models.

[52] used general structure of group methods of data handling (GMDH) modified by means of extreme learning machine (ELM) concept to develop a D model using 233 experimental data sets related to D to conduct training.

Although the machine learning AI techniques have shown great capacity for speed and accuracy in D predictions, there are still setbacks in their applications. The input data must be accurate, because the technique is highly susceptible to error that can lead to biased predictions resulting from biased training set. The selection of algorithm in machine learning is still a manual process. The main problem occurs in training and testing of data and because huge data is involved, removing error is nearly impossible. Such errors take plenty of time to resolve if detected. Machine learning technique can review large volumes of data and discover specific trend and patterns not apparent to human, but rivers and streams are so dynamic that hydraulic feature can change in matter of minutes. Machine learning AI depend on high-quality data for training accurate models. Data collection and processing which are crucial are time-consuming, laborious and expensive. Moreover, the technique is highly skilled and expensive. This makes it not readily available to engineers outside of the employ of government and big companies in the developed world. AI does not recognize causes and effects that may affect obtained data which may require changing on decisions in a short duration. Mathematical and empirical models are still the more useful, convenient, simple, unambiguous, handy tool for quick and easy computation of dispersion coefficient.

EXPERIMENTAL SETUP

Six S shaped out-door channels of different number of meanders were made from the overall curvilinear lengths of the channel and straight length of each was $15.0m$. All channels were constructed on approximately equal slope. Channel shape was basically rectangular at the beginning and end but curved at the various meandering sections. Channel width varied from $0.2m$ to $0.5m$ while depth variations ranged between $0.2m$ to $0.3m$. The channel was allowed to develop for some time and then uniform flow was maintained before the tracer was introduced and while samples were taken.

All channels were linked to the smaller reservoir which supplied water to them through a $75mm$ Ø pipe fitted with ball valve to control flows. This also regulated velocity of flow once channel was fully developed. Water was supplied to reservoir A from a borehole drilled for that purpose. Reservoir A when half full was made to fill reservoir B from which the channels were developed. This way constant head was maintained in both reservoirs B and the channels. All four supply sources were run concurrently (i.e the borehole-tank A- tank B channel) to maintain a constant head. The channels were constructed of sandcrete walls and floor and sloped. The floor was covered with a layer of river sand. Grass and mold were allowed to grow on the floor and walls before the experiments were performed. These were meant to, as much as possible, mimic natural river conditions. The channels were models of the Kaani river stretch from Yeghe in Gokana local council ($4^0$39’35”N $7^0$16’57”E to Wiiyaakare in Khana local council ($4^0$42’N $7^0$21’E) all in the Nigeria’s south-south state of

Rivers. The river has some bends along the stretch that are difficult to measure. The velocities, and hence water depth, were varied arbitrarily for the purpose of this study. The study area experiences frequent rains between March and late September causing frequent variations in velocity. The sand mining activities upstream also fluctuates velocity. The data obtained in these experiments can therefore compare favorably with those of natural streams of similar features.

Salt solution was made by thoroughly mixing 40gramms of sodium chloride (common salt) with 200ml of water. This was used as the tracer material. This solution was introduced at 1.5m away from the channel feed point to reduce the effect of turbulence generated by the at the point of supply from tank B. Samples were collected at the channel outlet. Some time in seconds corresponding to twice the detention time was allowed to elapse before the commencement of sample collection. Sampling times were predetermined at regular intervals, constituting constant-distance, variable-time method of sampling.

The models indicate the relationship between dispersion coefficient and the hydraulic and geometric parameters that affect dispersion in rivers. The models show better prediction ability than the most recent models adjudged as the most reliable including [12], [21], [30] as is evident in their correlation values and other statistical measures. Peculiar to this model is the inclusion of number of meanders (N), channel sinuosity (Si) and ratio of radius of curvature to Hydraulic radius. Radius of curvature has featured in many transverse mixing coefficient models but not in longitudinal dispersion coefficient because researchers have thought of curvature as affecting mixing only. Only a few models have sinuosity seen as input in determining D. The ratio (Rc/Rh) is entirely new having never featured in any model. These parameters have effects on dispersion as they well correlate with DM. The coefficient of correlation of Rc/Rh shows dispersion in rivers depend not only on radius of curvature but also on its hydraulic radius. Hydraulic radius incorporates the width and depth of flow and invariably represents the channel shape factor comprising of the bed shape factor and side wall effect, two aspects of channel features that have been difficult to determine.

RESULT AND DISCUSSIONS

Velocity profile has profound influence on the fundamental mechanisms of dispersion. Studies have therefore concentrated on the stream-wise depth averaged velocity distribution in the development of models for dispersion coefficient. Many others have relied on Fischer's triple integral as basis for the models [24], [35], [37], [25], [6]. All of these models either overestimate or underestimate D. All the equations (mathematical and empirical) reviewed lack reproducibility and predictive accuracy, two essential elements in science [16]. As already seen some of the models considered have inconsistencies [31] and all of them are too way off measured values of D, and none can be used with confidence in any particular reach before calibration and verification.

The poor predictive performance and irreproducibility of these models can be seen in Table.1 [12] in which only a few predicted values of D by available models were up to 80% of the measured value. None of the models have up to 32% predictive ability. The poor performance of these equations does not stem only from the fact that processes contributing to dispersion are not yet understood [12], but also from no inclusion of certain factors that are known to influence dispersion. For example, none of the models (except [12], [38]) showed improved and predictive ability when channel sinuosity is considered as input. Thus, all the equations considered so far are grossly inadequate for predicting D in sinuous rivers.

Table 1. Measured and predicted values of dispersion coefficient by various models

	Measured Value	Sahay (2013)	Fischer (1975)	Liu (1977)	Seo and Cheong (1998)	Kashefipour And Falconer (2002)	Deng et al (2001)	SahayandDutta (2009)	TayfurandSingh (2005)
Antietam River Md	20.9	28.6	5.1	7.4	20.2*	15.2	15.0	13.7	26.8
Bear Creek, Colo	2.8	25.1	7.3	33.6	52.2	29.1	28.1	39.1	39.2
Chattahooch River, Va	88.9	108.2	127.9	168.6	169.1	82.1*	168.8	147.1	77.6
Clinch River, Va	10.7	15.3	26.4	37.8	27.6	11.5*	28.5	25.8	26.9
Clinch River, Va	36.9	70.9	52.6	56.2	139.6	104.1	118.3	97.8	76.6
Conococh Creek, Md	53.3	63.3	88.2	66.5	96.3	58.8	93.2	78.2	43.0
John Day, Ore	13.9	41.3	86.4	72.9	83.3	44.8	81.8	11.2	45.2
John Day, Ore	65.0	12.4	19.3	32.6	116.7	97.9	71.1	73.2	77.2
Missourri River	89.2	897.1	4119.6	776.0	1317.3	990.5	952.1	1074.7	763.4
Monocacy River, Md	37.8	19.2	61.7	90.3	27.1	7.6	28.2	31.7	27.1
Monocacy River, Md	41.4	17.8	74.6	188.1	23.5	4.2	25.8	33.4	31.4
Powel River, Tenn	15.5	15.5	5.4	23.5	9.9	2.9	9.9	10.3	25.3
Sabina River, La	308.9	397.3	2535.1	477.0	718.7	512.3	509.2	603.8	346.6*
Sabina River, Texas	12.8	9.5	2.0	5.0	5.2	2.4	4.6	4.4	21.6
Tabgipahoa River, La	44.0	30.9	142.1	33.2	39.2	24.5	28.7	34.7	26.5
Wind/Big River, Wyo	41.8	94.8	229.6	186.8	159.6	16.0	156.7	147.0	59.7
		2/16 =5%	0/16 =0%	1/16 2.5%	1/16 2.5%	3/16 18.75%	2/16 5%	0/16 =0%	1/16 2.5%

Paired T-tests at alpha 0.05 and 0.1 levels of confidence were performed for results obtained from the models in table 1. The result showed that [21] and [25] have significant difference at 0.05 and 0.1 levels of significance while [12], [24], [35], [19], [38], [23] have no significant difference at 0.05 and 0.1 levels of significance between the measured and predicted dispersion coefficients. There is therefore similarity between the measured and the predicted result for all models except for [25], [21]. From the table below, [12] has the closest relationship with the measured at T_{cal} of 0.79 followed by [19] with T_{cal} 1.26, [38] also at 1.26, then [24] with 1.57, [35] with 1.94, [23] with 2.20 and [25] at 2.7 respectively.

The implication of these results is that all these models, except [21] and [25], show ability in determining longitudinal dispersion coefficient for rivers but lack accuracy. The reason for the inaccuracy is attributed to neglecting some of the irregularities that affect dispersion. Such irregularities include bends and the centrifugal forces that are generated in them during flow and perhaps the number of such bends. [12] suggests that the disparities are because the processes that lead to determination of dispersion coefficient are not yet well understood.

Table 2. Paired T-test result at 5% level of confidence and coefficient of correlation between measured and predicted values.

Name	T_{cal}	T_{tab}	R^2
Deng et al (2001)	2.70	2.13	0.949
Sahay and Dutta (2009)	2.20	2.13	0.1185
Liu (1977)	1.94	2.13	0.8914
Sahay (2013)	0.79	2.13	0.9827
Fischer (1975)	1.57	2.13	0.932
Seo an Cheong (1998)	2.47	2.13	0.9559
Kashiefipour& Falconer (2002)	1.26	2.13	0.9623
Tayfur and Singh (2005)	1.26	2.13	0.9845

Fig. 1 is the bar chart of the percentage of calculated results of dispersion coefficient for the models in table 1 which was up to eighty percent of the measured values. Kashefipour and Falconer has 3 results reaching 80% of the measured amounting to 18.5% on the chart. The models by [24] , [23] have 0%, implying that no results from these models attained 80% of the measured values. [12] and [25] had 2 results (5 %) while [35] and [38] had 1 result (2.5%) each. These findings show the unreliability and inaccuracy of these models as they grossly underestimate dispersion coefficient, their methods of formulation notwithstanding. It proves further that the fundamentals processes in determining longitudinal dispersion coefficient are not yet well understood.

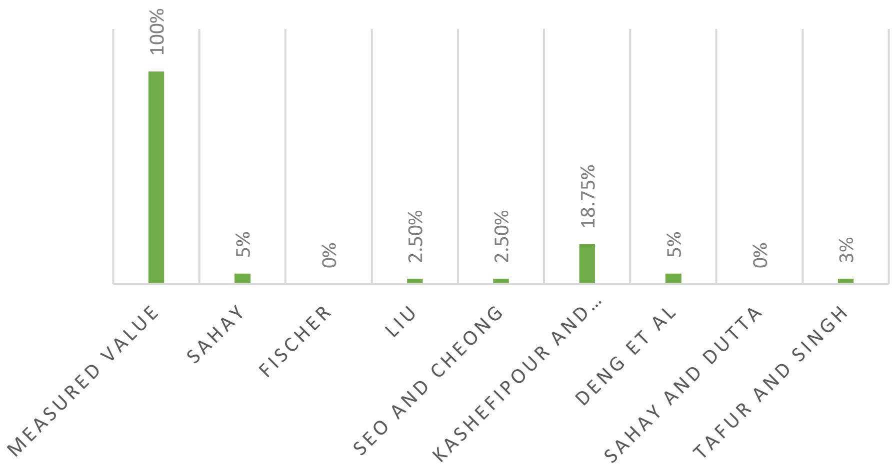

Figure 1. Percentage of predicted value that is up to 80% of measured value.

The plots of measured results regressed against predicted values (figs 2 – 8) show very good agreement with coefficient of correlation of more than 0.8 for the models except for [23]. This implies that the models represent correct methodology for obtaining longitudinal dispersion coefficient for the rivers that were measured, but that perhaps some of the fundamental factors that affect dispersion were not accounted for resulting in large disparity between measured and predicted results.

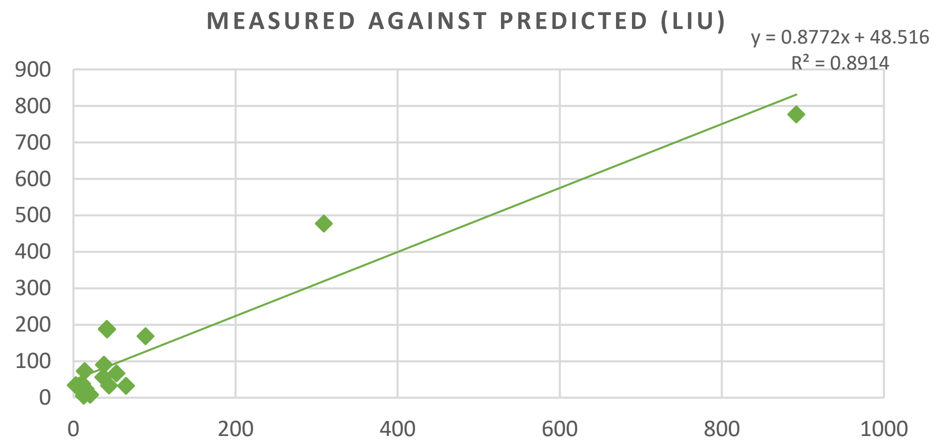

Figure 2. Plot of measured against predicted (Liu)

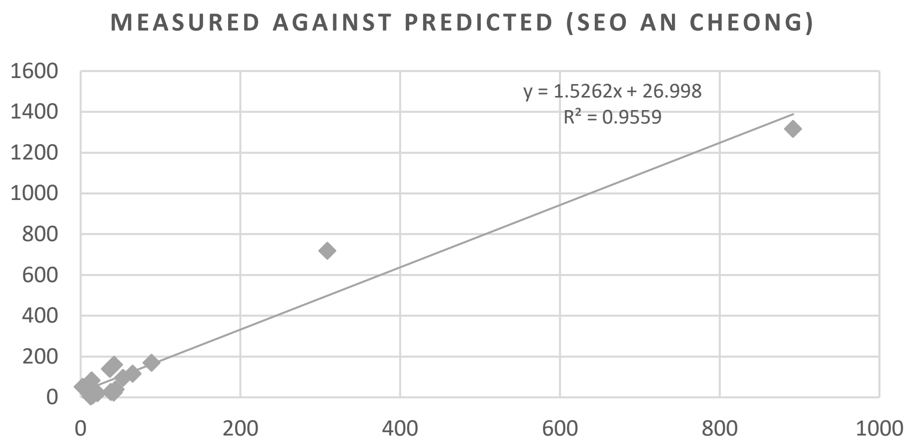

Figure 4. Plot of Measured against Predicted (Seo and Cheong)

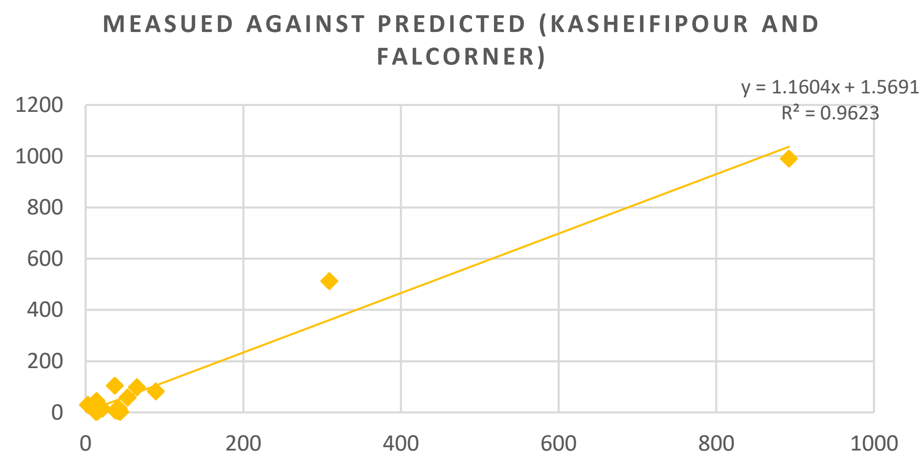

Figure 5. Plot of Measured against Predicted (Kasheifipour and Falcorner)

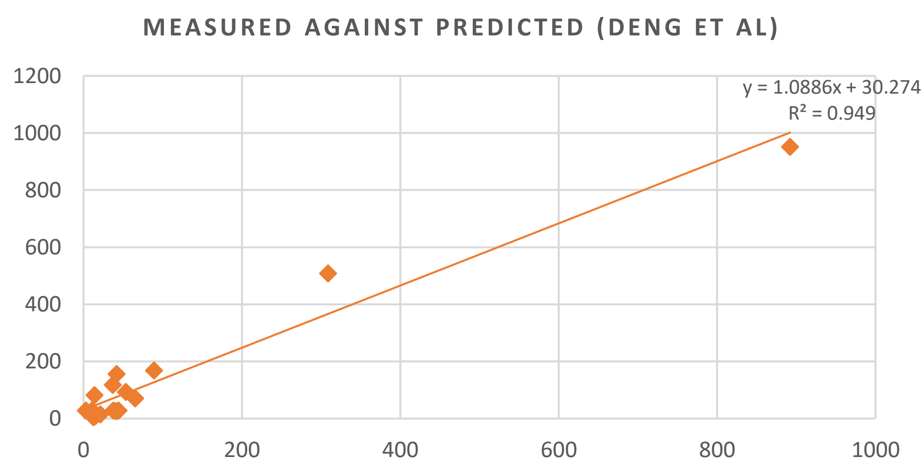

Figure 6. Plot of Measured against Predicted (Deng et al)

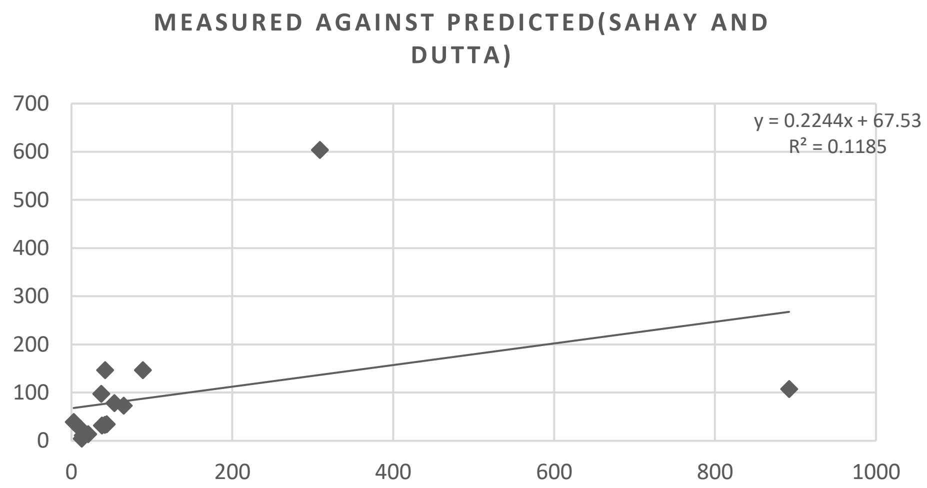

Figure 7. Plot of Measured against Predicted (Sahey and Dutta)

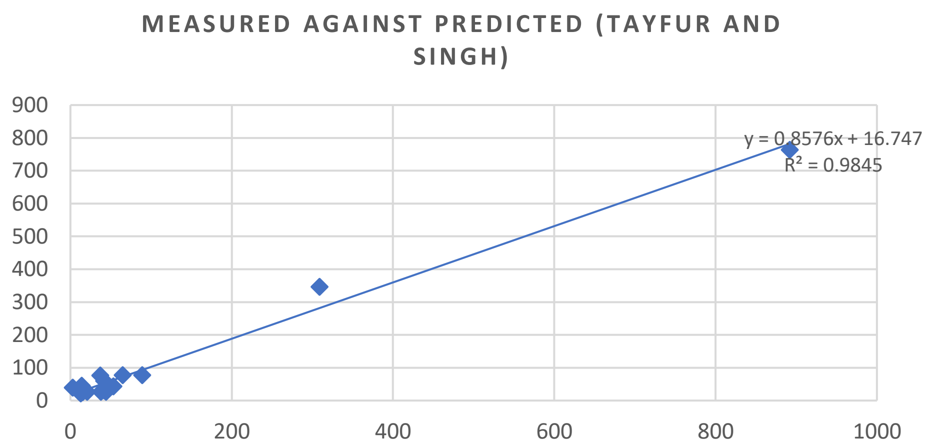

Figure 8. Plot of Measured against Predicted (Seo and Cheong)

Channel irregularities include sinuosity, radius of curvature, number of bends, and other geometric conditions. That these factors affect dispersion are evident [18], [25], [29]. These factors can be mathematically related to dispersion coefficient in the form:

$$D = f\,(B,H,R,N,S,R_H,U,U_*) \qquad (32)$$

The general form of such equation can also be written as

$$D=\propto \left(\frac{U}{U_*}\right)^a \left(\frac{B}{H}\right)^b (HU_*)^c \left(\frac{R_c}{R_h}\right)^d N^e S_i^f \qquad (33)$$

Where U= mean flow velocity, U_*=Shear velocity, B=width of channel, H=depth of channel, R_c =radius of curvature, N= number of meanders, S_i = sinuosity and R_H = Hydraulic radius
The Buckingham pi theorem method of dimensional analysis is a ready tool in formulating this form of equation.

A further research by current authors is in process as “Modeling Dispersion coefficient by use of Dimensional Analysis”. The investigation explains in detail how dimensional analysis was used to relate the parameters that affect dispersion and how reproducibility can be

achieved. The new parameters included in the new equations and related using dimensional analysis are (1) Hydraulic radius, R_H (2) Radius of curvature, R_C (3) Number of meanders, N.
These parameters, together with those usually found in the literature on this subject matter, were related by dimensional analysis. The models were calibrated using MATLAB and the GRG model of the Excel solver, though only the MATLAB results were used because they agreed better with the field result.
Tracer tests were conducted in a series of channels. The samples collected were analyzed in a chemical laboratory in Nigeria for tracer concentration which were used to obtain field result of dispersion coefficient by the Levenspiel and Smith (1957) method. These results were used in calibrating the equations.
The parameter N restricts the use of the equations to the confines of a meandering condition. N is a kind of "moderator" here. This implies that once the dispersion coefficient has been calculated for the meandering channel, applying the number N should bring it reasonably close to the field value. N is the number of bends (meanders) in the river. This way, reproducibility is achieved, as any river reaching a number of meanders between 2 and 6 has an equation associated with it and may not be used otherwise. There were several numbers of meanders (six of them in all) that were modeled, and they all gave better results than the other models compared.

CONCLUSION

This paper has demonstrated that there is a significant difference between measured and predicted values in the D models that are currently in use. The discrepancy stems from a number of factors, including the incomplete understanding of the dispersion process, the omission of some crucial hydrodynamic and geometric variables, and the derivation of some of these variables based on assumptions that do not align with the geometry and flow of rivers in their natural state. Including these variables in an equation will increase the accuracy and repeatability of predictions.
Because the mathematical models require sampling and laboratory analysis to determine concentration before they are applied, they are not useful as an on-the-spot assessment tool by the engineer, nor do they adequately describe the process. The moment method, which is based on the second moment, is unable to accurately determine D because it does not address the problem of the long tails and skew seen in c-t data plots. Although the more recent use of machine learning techniques is producing accurate results, there are still many errors due to missing data. Additionally, they are costly, highly skilled, and difficult for engineers to obtain, particularly in developing nations.

Data Availability Statement

All relevant data used for this research are included in the paper.

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CONFLICT OF INTEREST

There are no conflicts of interest between the authors and their institutions that could appear to have influenced the work presented in this publication.

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